Instructions: *Do* any five *of the following problems. Be sure to write the number for each problem you work out and write your name clearly at the top of each page you turn in for grading. You have three and a half hours.* Good luck!

1. Compute the fundamental group of S^2 with n points removed, where n is a positive integer. Also compute the fundamental group of $\mathbb{R}^3$ with the three coordinate axes removed.

2. Let $X = S^1 \vee S^1$, the wedge of two circles. Exhibit both a regular and an irregular 4-fold covering of X. Does X have an irregular 2-fold covering? Explain.

3. Let $p\colon (E, e_0) \to (B, b_0)$ be a covering map, where B and E are path-connected and locally path-connected. Let $H = p_*(\pi_1(E, e_0))$ be the image of the fundamental group of E in $\pi_1(B, b_0)$. Prove that the group of covering transformations of E is isomorphic to $N(H)/H$, where $N(H)$ is the normalizer of H.

4. Give an example of a space whose fundamental group is cyclic of order 13. Prove that its fundamental group is indeed cyclic of order 13.

5. Let X be path-connected and locally path-connected, and suppose that the fundamental group of X is finite. Prove that any continuous function $f\colon X \to S^1$ is null-homotopic.

6. Let A be a path-connected subspace of a space X, and $a_0 \in A$. Prove that the homomorphism $\pi_1(A, a_0) \to \pi_1(X, a_0)$ induced by inclusion is surjective if and only if every path in X with endpoints in A is path-homotopic to a path in A.

7. State the Seifert-van Kampen Theorem, and use it to find the fundamental group of the space $X \subset \mathbb{R}^2$ that is the union of two circles of radius 1 with centers $(-2, 0)$ and $(2, 0)$ and the interval $[-1, 1] \times \{0\}$.