Instructions: *Do any* **five** *of the following seven problems. Justify your statements and calculations. Be sure to write the number for each problem you work out and write your name clearly at the top of each page you turn in for grading. You have three and a half hours.* Good luck!

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1. Let $X = \{(x, y, z) \in \mathbb{R}^3 | x = 0;\ y = 0\} \cup \{(x, y, z) \in \mathbb{R}^3 | x^2 + y^2 = 1;\ z = 0\}$. Calculate the fundamental group of $\mathbb{R}^3 \setminus X$.

2. Show every continuous map from the real projective plane to a torus is homotopic to a constant map. Give an example of a continuous map from a torus to the real projective plane which is not homotopic to a constant map. Explain why your example is not homotopic to a constant map.

3. Show that a retract of a contractible space is contractible.

4. Let A be a path-connected subspace of a space X and $a_0 \in A$. Show that the inclusion induces a surjection from $\pi_1(A, a_0)$ to $\pi_1(X, a_0)$ if and only if every path in X with endpoints in A is path-homotopic to a path in A.

5. Let $X = S^1 \vee S^1$, the wedge of two circles. Give an example of a regular 3-fold cover of X. Give an example of an irregular 3-fold cover of X. Is there an irregular 2-fold cover of X? Note the terminology used by Hatcher for regular and irregular is normal and non-normal.

6. Derive a presentation of the fundamental group of a closed surface of genus two. Show this group is non-abelian.

7. a) Complete the following definition: Two topological spaces X and Y are homotopy equivalent if .....

   b) Let X, Y, Z be topological spaces. Prove that if X is homotopy equivalent to Y, and Y is homotopy equivalent to Z, then X is homotopy equivalent to Z.