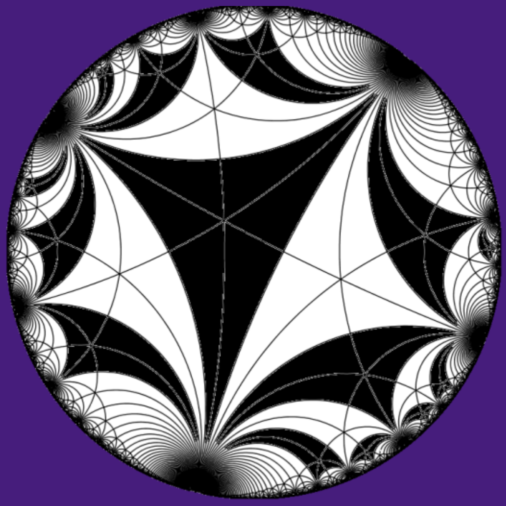
Southern Regional
Number Theory Conference

Modular Forms, Modular Curves, and Hypergeometric Functions

April 13-14, 2019
Louisiana State University,
Baton Rouge, Louisiana

Plenary Speakers

Luca Candelori	*Wayne State University*
William Chen	*McGill University*
Jenny Fuselier	*High Point University*
Victor Hugo Moll	*Tulane University*
Armin Straub	*University of South Alabama*
Yilong Wang	*Louisiana State University*



Organizers: Jerome Hoffman, Ling Long, Karl Mahlburg, Liem Nguyen, Bao Pham, Fang-Ting Tu

LSU
Department of Mathematics

https://www.math.lsu.edu/nt2019

April 13		April 14	
8:30 - 8:50	Registration		
8:50 - 9:00	*Welcoming address*		
9:00 - 10:00	Luca Candelori	9:00 - 10:00	Jenny Fuselier
10:00 - 10:30	Morning Tea	10:00 - 10:30	Tea + Photo
10:30 - 11:30	Yilong Wang	10:30 - 11:00	Madeline Dawsey
11:30 - 12:00	Shashika Petta Mestrige	11:00 - 11:30	Po-Han Hsu
12:00 - 12:30	Michael Allen	11:30 - 12:30	Armin Straub
12:30 - 2:00	Lunch		
2:00 - 3:00	William Chen		
3:00 - 3:30	Tea + Photo		
3:30 - 4:30	Victor Moll		
4:30 - 5:00	Wei-Lun Tsai		
5:00 - 5:30	Oguz Gezmis		
5:30 - 6:00	Changningphaabi Namoijam		
6:30 - 8:30	Dinner		

*The lecture will be held in Lockett 233

April 13, 2019

Integrality properties of the Weil Representation of a finite quadratic module

Luca Candelori, Wayne State University

The Weil representation of a finite quadratic module is an essential tool in studying the transformation laws of theta functions. This complex, finite-dimensional representation factors through a finite quotient of the metaplectic cover of the modular group and it has a canonical basis of delta functions. With respect to this basis, the matrix entries of the Weil representation lie in a cyclotomic field extension. In this talk we prove that after a suitable change of basis the matrix entries can be taken to lie in the ring of integers of the cyclotomic extension. For cyclic quadratic modules of even prime-power order, we exhibit a canonical choice for such an integral basis. The original motivation behind this question was the development of Integral Topological Quantum Field Theory by Gilmer and Massbaum. We discuss the connections with this integral theory, as well as with integral TQFTs afforded by more general modular tensor categories. This is joint work with Richard Ng and Yilong Wang at LSU, and with Yilong Wang and Shaul Zemel at Hebrew University.

On higher Gauss sums of modular categories

Yilong Wang, Louisiana State University

Modular tensor categories are categorical generalizations of nondegenerate quadratic forms, and the notion of Gauss sums can be analogously defined. In this talk, we will discuss the arithmetic properties of the higher Gauss sums and the Witt invariance of the associated higher central charges. This talk is based on a joint work with Andrew Schopieray and Siu-Hung Ng.

Ramanujan Congruences for Infinite Family of Partition Functions

Shashika Petta Mestrige
Louisiana State University

The partition function $p_{[1^c\ell^d]}(n)$ can be defined using the generating function,

$$\sum_{n=0}^{\infty} p_{[1^c\ell^d]}(n)q^n = \prod_{n=1}^{\infty} \frac{1}{(1-q^n)^c(1-q^{\ell n})^d}.$$

In this paper, we prove infinite families of congruences for the partition function $p_{[1^c\ell^d]}(n)$ modulo powers of ℓ (where ℓ is a prime ≤ 13) for any integers c and d (with $c > 0$ and $d \geq -2$ for $\ell \neq 11$). We use Hecke operators, explicit basis of the vector space of modular functions of the congruence subgroup $\Gamma_0(\ell)$ and work of Atkin and Gordon on proving congruences for the partition function $p_{-k}(n)$.

Existence of η-quotients of squarefree levels

Michael Allen, Oregon State University

In *The Web of Modularity*, Ono observes that every modular form for $\mathrm{SL}_2(\mathbb{Z})$ can be written as a rational expression in η-quotients, and poses the question of which graded rings of modular forms can be similarly generated by η-quotients. Towards answering this question, we give necessary and sufficient conditions for the existence of η-quotients in modular form spaces of weight k and level N. Using these, we exhibit an infinite family of levels N such that the ring $M(\Gamma_0(N))$ is not generated by η-quotients.

Nonabelian level structures and noncongruence modular forms

William Chen, McGill University

I will describe what is known about nonabelian level structures for elliptic curves, how they influence the arithmetic of the Fourier coefficients of noncongruence modular forms, and some interesting questions that come up. As time allows, I will describe the particular case of metabelian level structures, where more can be said.

p-adic Properties of Interesting Sequences

Victor Moll, Tulane University

Given an integers x and a prime p, the p-adic valuation of x, denoted by $\nu_p(x)$ is the highest power of the prime p dividing x. The structure of the sequence $\{\nu_p(x_n)\}$, for several sequences of number theoretical interest will be discussed.

Arithmetic statistics of canonical Hecke L-functions

Wei-Lun Tsai, Texas A&M University

The canonical Hecke characters in the sense of Rohrlich form a set of algebraic Hecke characters with important arithmetic properties. For example, the central values of their corresponding L-functions are related to ranks of Gross's elliptic $\mathbb{Q}$-curves. In this talk, we explain how non-trivial bounds for ℓ-torsion in class groups of number fields can be used to prove that for an asymptotic density of 100 percent of CM fields E within certain general families, the number of canonical Hecke characters of E whose L-function has a nonvanishing central value is $\gg |disc(E)|^{\delta}$ for some absolute constant $\delta > 0$. This is joint work with B. D. Kim and Riad Masri.

Multiple Polylogarithms over Tate Algebras

Oguz Gezmis, Texas A&M University

Multiple Zeta values and multiple polylogarithms have been studied recently due to its occurrence in different fields of mathematics such as knot theory and mathematical physics. In 2004, Thakur introduced multiple zeta values on function fields and in 2014 Chang discovered the relation between multiple polylogarithms and multiple zeta values in this content. Recently Pellarin introduced multiple zeta values over Tate algebras and moreover he proved a sum-shuffle formula among them. In this talk we introduce multiple polylogarithms over Tate algebras and relate them to multiple zeta values defined by Pellarin. This is a joint work with Federico Pellarin.

Hyperderivatives of Periods of Drinfeld Modules and Transcendence

Changningphaabi Namoijam, Texas A&M University

We investigate hyperderivatives of periods of Drinfeld modules by studying an Anderson t-module whose periods involve hyperderivaties of periods of the given Drinfeld module. We further investigate transcendence of periods using Papanikolas' results on transcendence degree of the period matrix of a t-motive and dimension of its Galois group.

April 14, 2019

Hypergeometric functions over finite fields

Jenny Fuselier, High Point University

In this talk, we introduce hypergeometric functions over finite fields, originally due to Greene, Katz, and McCarthy. We provide an overview of these functions and then highlight ways they relate to point-counting, traces of Hecke operators, and supercongruences. We also present a systematic approach for translating some classical hypergeometric identities and evaluations to the finite field setting by an explicit dictionary.

Moonshine for Finite Groups

Madeline Dawsey, Emory University

Weak moonshine for a finite group G is the phenomenon where an infinite dimensional graded G-module

$$V_G = \bigoplus_{n \gg -\infty} V_G(n)$$

has the property that its trace functions, known as McKay-Thompson series, are modular functions. Recent work of Dehority, Gonzalez, Vafa, and Van Peski established that weak moonshine holds for every finite group. Since weak moonshine only relies on character tables, which are not isomorphism class invariants, non-isomorphic groups can have the same McKay-Thompson series. We address this problem by extending weak moonshine to arbitrary width $s \in \mathbb{Z}^+$. Namely, for each $1 \leq r \leq s$ and each irreducible character χ_i, we employ Frobenius' r-character extension $\chi_i^{(r)} : G^{(r)} \to \mathbb{C}$ to define McKay-Thompson series of $V_G^{(r)} := V_G \times \cdots \times V_G$ (r copies) for each r-tuple in $G^{(r)} := G \times \cdots \times G$ (r copies). These series are modular functions. We find that *complete* width 3 weak moonshine always determines a group up to isomorphism. Furthermore, we establish orthogonality relations for the Frobenius r-characters, which dictate the compatibility of the extension of weak moonshine for V_G to width s weak moonshine.

On Selberg's Central Limit Theorem for Dirichlet L-Functions

Po-Han Hsu, Louisiana State University

In probability theory, the central limit theorem roughly tells us that any sequence of random variables converges to a normal distribution. In 1946, Selberg proved that $\log|\zeta(\frac{1}{2}+it)|$ will possess a distribution approximate to the normal distribution when t is large, which is extended to a large class of L-functions by himself and is nowadays called the Selberg's central limit theorem. In 2017, Radziwiłł and Soundararajan gave a new proof of the Selberg's central limit theorem for $\log|\zeta(\frac{1}{2}+it)|$.

In this talk, we will discuss how to extend the method developed by Radziwiłł and Soundararajan to Dirichlet L-functions. If time permits, we will also discuss the independence property of random variables arising from the associated L-functions.

Interpolated sequences and critical L-values of modular forms

Armin Straub, University of South Alabama

It is well-known that the Apéry sequences which arise in the irrationality proofs for $\zeta(2)$ and $\zeta(3)$ satisfy many interesting arithmetic properties and are related to p-th Fourier coefficients of modular forms. We discuss how these connections persist in the general context of sequences associated to Brown's cellular integrals.

Moreover, Zagier recently expressed an interpolated version of the Apéry numbers for $\zeta(3)$ in terms of a critical L-value of a modular form of weight 4. We extend this evaluation in two directions. We first show that interpolations of Zagier's six sporadic sequences are essentially critical L-values of modular forms of weight 3. We then establish an infinite family of evaluations between interpolations of leading coefficients of Brown's cellular integrals and critical L-values of modular forms of odd weight.

This talk is based on joint work with Robert Osburn. Earlier joint work with Dermot McCarthy and Robert Osburn is featured as well.