QM Abelian Varieties, Hypergeometric Character Sums, and Modular Forms

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This is a report of work in progress with Winnie Li, Ling Long, and Fang Ting Tu. The theme is to relate hypergeometric character sums to modular forms in an interesting family of cases. These arise from arithmetic triangle groups related to the quaternion algebra B over $\mathbb{Q}$ with discriminant $D = 6$. Quotients of the upper half plane by the units in these algebras give rise to Shimura curves, which are moduli spaces for 2-dimensional abelian varieties with quaternion multiplication (QM).

In the talk, I will explain the geometric background of this problem, especially the Eichler-Shimura theory relating modular forms to parabolic cohomology, both in the complex-analytic and in the ℓ-adic etale setting. The key result, due to Kuga-Shimura, computing the zeta functions of the fiber spaces of abelian varieties in terms of Hecke polynomials, allows one to relate these hypergeometric character sums to traces of Hecke operators on spaces of modular forms.