New Expansions for Mersenne Primes

Moustafa Ibrahim

This talk highlights some of my new results for the Lucas-Lehmer primality test for Mersenne numbers and also develops the Euclid-Euler theorem for the even perfect numbers. For examples, we show the following fresh results:

Theorem 1 *Lucas-Lehmer-Moustafa (Version 1)*
Given prime $p \geq 5$, $n := 2^{p-1}$. The number $2^p - 1$ is prime **if and only if**

$$2\,n - 1 \quad \Big| \quad \sum_{\substack{k=0,\\ k \text{ even}}}^{\lfloor \frac{n}{2} \rfloor} (-1)^{\lfloor \frac{k}{2} \rfloor} \frac{\prod_{\lambda=0}^{\lfloor \frac{k}{2} \rfloor - 1} n^2 - (4\lambda)^2}{k!}.$$

Theorem 2 *Lucas-Lehmer-Moustafa (Version 2)*
Given prime $p \geq 5$. $2^p - 1$ is prime **if and only if**

$$2^p - 1 \quad \Big| \quad \sum_{\substack{k=0,\\ k \text{ even}}}^{\lfloor \frac{n}{2} \rfloor} \phi_k(n)$$

where $n := 2^{p-1}$, $\phi_k(n)$ are defined by the double index recurrence relation

$$\phi_k(m) = -4\,\phi_{k-1}(m-2) - \phi_k(m-4)$$

and the initial boundary values satisfy

$$\phi_0(0) = 2, \quad \phi_1(0) = 0, \quad \phi_1(2) = -4, \quad \phi_k(0) = \phi_k(2) = 0 \quad \forall k \geq 2$$

and

$$\phi_0(n) = \begin{cases} +2 & n \equiv \pm 0 \pmod 8 \\ 0 & n \equiv \pm 2 \pmod 8 \\ -2 & n \equiv \pm 4 \pmod 8 \end{cases}$$

Theorem 3 *Lucas-Lehmer-Moustafa (Version 3)*
Given prime $p \geq 5$. $2^p - 1$ is prime **if and only if**

$$2^p - 1 \quad \Big| \quad \sum_{\substack{k=0,\\ k \text{ even}}}^{\lfloor \frac{n}{2} \rfloor} \phi_k(n)$$

where $n := 2^{p-1}$, $\phi_k(n)$ are defined by the double index recurrence relation

$$\phi_k(n) = -\frac{n^2 - (2k-4)^2}{k\,(k-1)}\;\phi_{k-2}(n)$$

where

$$\phi_0(n) = 2.$$