Wannabe Modularity, the Shintani-Faddeev Cocycle, and Stark Units

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We define wannabe modular forms, functions on the upper half-plane transforming under the action of the modular group by a generalized factor of automorphy. We define a related notion of wannabe Jacobi forms, and we show that the q-Pochhammer symbol is a meromorphic wannabe Jacobi form. Its factor of automorphy is the Shintani-Faddeev cocycle, an $SL_2(\mathbb{Z})$ -parametrized family of functions generalizing Shintani's double sine function and Faddeev's noncompact quantum dilogarithm. We relate real multiplication values of the Shintani-Faddeev cocycle to exponentials of certain derivative L-values, conjectured by Stark to be algebraic units generating abelian extensions of real quadratic fields.