New results on Polynomials in Difference Sets

Alex Rice

It is a well known result, established independently by Sárközy and Furstenberg, that a set of integers with positive upper density must contain two distinct elements that differ by a perfect square. Here we will discuss the best-known quantitative improvements of this result, as well their extension to the largest possible class of univariate polynomials. Further, we discuss even better upper bounds for a large class of multivariate polynomials. In both settings, we utilize a sieve as a bridge to algebraic geometry, gaining access to optimal exponential sum estimates over finite fields.