

A Public Lecture of SRNTC



delivered by Professor Victor H. Moll, Tulane University



The Evaluation of Integrals. A Collection of Questions

Lockett 9, Saturday, March 21, 5:00-6:00 p.m.
Refreshments at 4:30 p.m. around Lockett 9

The question of evaluation of definite integrals is one of the basic questions of elementary Mathematics.

In this talk the author will present a collection of problems that have appeared in the attempt to evaluate definite integrals. These will include some problems in Number Theory such as *what is the power of 2 which divides factorials and binomial coefficients* or *when is the central binomial coefficient exactly divisible by 2?* This connection comes from Wallis' formula

$$\int_0^{\infty} \frac{dx}{(x^2+1)^{n+1}} = \frac{\pi}{2^{2n+1}} \binom{2n}{n}$$

There are also some problems in Dynamical Systems such as *describe the behavior of the map*

$$x_n = \frac{n + x_{n-1}}{1 - nx_{n-1}}$$

with $x_1=1$. It is curious that the first 4 values of x_n are integers. It has been conjectured that this never happens again. There are also examples of Taylor series: it is a remarkable fact that the coefficients in the expansion of

$$h(x) = \sqrt{a + \sqrt{1+x}}$$

are related the integral

$$\int_0^{\infty} \frac{dx}{(x^4 + 2ax^2 + 1)^n}$$

A theorem of Ramanujan gives the connection. It is a classical result that Taylor coefficients of special functions have interesting arithmetic properties. For instance, the Bernoulli numbers, appearing in the expansion of $\tan(x)$, are connected to Diophantine equations.

The talk will present a tour of these ideas and it will accessible to undergraduate students.