Integer Partitions and Group Characters

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Inspired by Hilbert's 12th problem, G. R. Robinson and J. G. Thompson studied the number fields obtained from character values of alternating groups. For a finite group G, let $K(G)$ denote the field generated over $\mathbb{Q}$ by its character values. For $n > 24$, they proved that

$$K\left(A_n\right) = \mathbb{Q}\left(\left\{\sqrt{p^*}\ :\ p \leq n \text{ an odd prime with } p \neq n-2\right\}\right),$$

where $p^* := (-1)^{\frac{p-1}{2}}p$. Confirming a conjecture of John Thompson, we show that arbitrary suitable multiquadratic fields are similarly generated by the values of A_n-characters restricted to elements whose orders are only divisible by ramified primes. Our proof makes use of partitions of integers into distinct parts which are squares of π-numbers. Extending Thompson's conjecture, we also identify the fields generated by characters of finite linear groups. This is joint work with Ken Ono and Ian Wagner.