

Some basic techniques in analysis

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Question 1. Can you find the limit

$$\lim_{x \rightarrow 0} \frac{x^2 - \sin(x^2)}{x^4(1 - \cos x)} ?$$

Try to compute this limit by using L'Hospital's rule. What's happen?

Question 2. Does the series

$$\sum_{k=1}^{\infty} \frac{\sin \sqrt{k}}{k}$$

converge?

Can you apply certain tests of convergence of series you know to answer the above question?

Problems

Problem 1. Find the limit

$$\lim_{x \rightarrow 0} \frac{d^4}{dx^4} \left(\frac{x}{\sin x} \right).$$

Problem 2. Show that the limit

$$\lim_{n \rightarrow \infty} \left(\frac{\sin \frac{\pi}{n+1}}{1} + \frac{\sin \frac{2\pi}{n+1}}{2} + \dots + \frac{\sin \frac{n\pi}{n+1}}{n} \right)$$

is positive.