14.1 Graphs, Paths, and Circuits

# Objective 1: Understand relationships in a graph

A **graph** consists of a finite set of points called **vertices** and line segments or curves called **edges** that start and end at vertices. The singular of vertices is **vertex.** An edge that starts and ends at the same vertex is called a **loop**.

A graph has vertices A, B, C, and D. The vertices are connected by edges A D, A C, B D, and B C, and C C. Edges A C and B D cross, but the point of crossing is not a vertex. Edge C C begins and ends at the same point. So, it is a loop.


Two graphs are **equivalent** if they have the same number of vertices connected to each other in the same way. The placement of the vertices and the shapes of the edges are unimportant.

Graphs a and b. Each graph has vertices A, B, C, and D, and edges A B, B C, and C D. In graph a, the vertices form the corners of a square. The relative positions of the vertices are as follows. A, bottom left. B, top right. C, bottom right. D, top left. As a result, edges A B and C D cross. In graph b, vertices A, B, C, and D are positioned from top left to bottom right, so that the edges form a single falling curve.


# Objective 2: Model Relationships Using Graphs

In the early 1700’s, the city of Königsberg, Germany, was located on both banks and two islands of the Pregel River. The figure below shows that the town’s sections were connected by seven bridges.

The city map can be modeled by a graph using vertices to represent the land masses and edges to represent the bridges.

The Konigsberg bridge map. The map shows 2 islands in a river. 7 bridges connect the islands to the banks and to each other. The first island is connected to the left bank by 2 bridges, and it is connected to the right bank by 2 bridges. The second island is connected to the left bank by 1 bridge, and it is connected to the right bank by 1 bridge. A single bridge also connects the islands to each other.


A graph can be created to model the borders between New England states by using vertices to represent the states and edges to represent common borders.

A map of New England. Maine is the most northeast and borders New Hampshire on its western side. New Hampshire borders Maine, Vermont on its western side, and Massachusetts on its southern side. Vermont borders Massachusetts on its southern side and New Hampshire. Massachusetts borders New Hampshire and Vermont and on its southern side Connecticut and Rhode Island. Connecticut borders Massachusetts on its northern edge and Rhode Island on its east. 


# Objective 3: Understand and use the vocabulary of graph theory

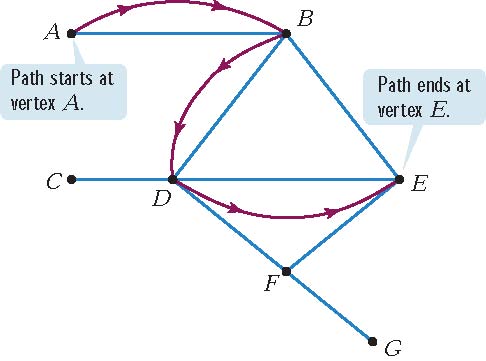
The **degree of a vertex** is the number of edges at that vertex. Since a loop connects a vertex to itself, that loop contributes 2 to the degree of the vertex. On a graph, the degree of each vertex is found by counting the number of line segments or curves attached to the vertex.

A vertex with an even number of edges attached to it is an **even vertex**. A vertex with an odd number of edges attached to it is an **odd vertex**.

Two vertices in a graph are said to be **adjacent vertices** if there is at least one edge connecting them. It is helpful to think of adjacent vertices as *connected* vertices.

A graph has vertices A, B, C, D, and E. The edges are A B, A B, A C, A D, A E, B D, C E, and E E. Vertex A has 5 edges so the degree is 5. Vertex B has 3 edges so the degree is 3. Vertex C has 2 edges so the degree is 2. Vertex D has 2 edges so the degree is 2, and vertex E has 2 edges and a loop contributing degree 2, so the total degree for E is 4.

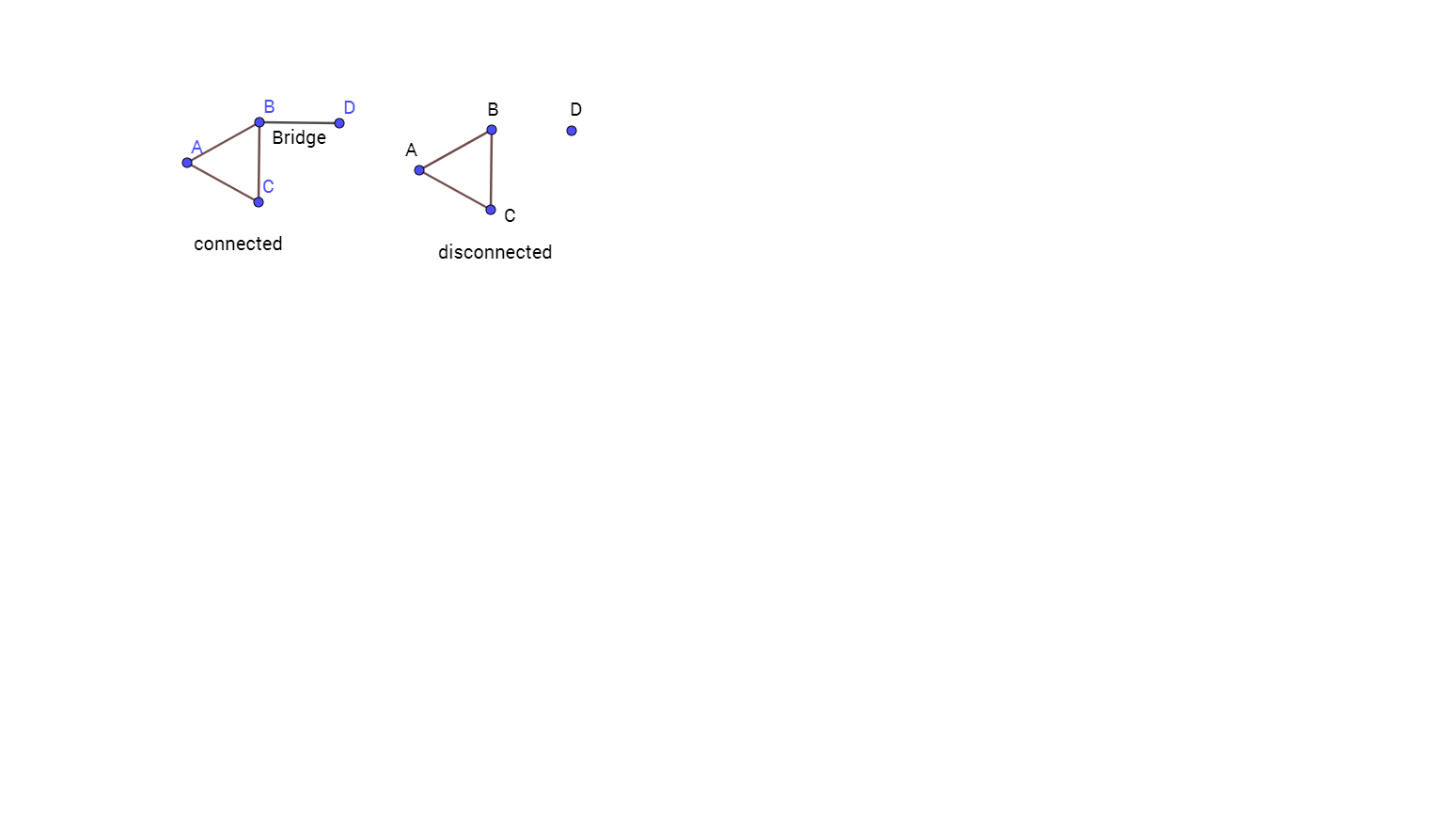
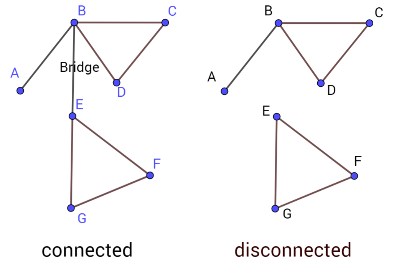

A **path** in a graph is a sequence of adjacent vertices and the edges connecting them. Movement along a path can be described by naming adjacent vertices. Although a vertex can appear on the path more than once, **an edge can be part of a path only once**. A path does not have to include every vertex and every edge of a graph.



A **circuit** is a path that begins and ends at the same vertex. Every circuit is a path, but not every path is a circuit, because not every path ends at the same vertex where it starts.

A graph has vertices A to G. The edges are A B, B D, B E, C D, D E, D F, E F, and F G. A path starts at B and passes through D, F, and E, before returning to B. The path starts and ends at B. So, it is a circuit.


A graph is **connected** if there is at least one path that connects any two vertices. Visually this means that a graph is connected if it consists of one piece. If a graph is not connected, it is said to be **disconnected**. A disconnected graph is made up of connected pieces that are called the components of the graph. A **bridge** is an edge that, if removed from a connected graph, would leave behind a disconnected graph.

 Note that the city bridges in the Konigsberg bridge map are simply edges of the graph where the land masses are vertices. None of these edges are bridges that would make the difference between a connected and a disconnected graph.