

Topic 16: Maximum and minimum values of a function

Find all critical numbers and determine whether each represents a local maximum, local minimum or neither.

1. $f(x) = x^3 - 3x + 1$

2. $f(x) = x^4 - 3x^3 + 2$

3. $f(x) = \sin x \cos x$ $[0, 2\pi]$

4. $f(x) = \frac{x}{x^2 + 1}$

5. $f(x) = \frac{1}{2}(e^x + e^{-x})$

6. $f(x) = x^{4/3} + 4x^{1/3} + 4x^{-2/3}$

Find the absolute extrema of the given function on the indicated interval.

7. $f(x) = x^4 - 8x^2 + 2$, $[-3, 1]$

8. $f(x) = \sin x + \cos x$, $[0, 2, \pi]$

Answers

1) $f(-1) = 3$, local maximum; $f(1) = -1$, local minimum

2) $x = 0$, neither; $f\left(\frac{9}{4}\right) = \frac{-1675}{256}$, local minimum

3) $f\left(\frac{\pi}{4}\right) = f\left(\frac{5\pi}{4}\right) = \frac{1}{2}$, local maxima; $f\left(\frac{3\pi}{4}\right) = f\left(\frac{7\pi}{4}\right) = -\frac{1}{2}$, local minima

4) $f(-1) = -\frac{1}{2}$, local minimum; $f(1) = \frac{1}{2}$, local maximum

5) $f(0) = 1$, minimum

6) $f(-2) = 0$, $f(1) = 9$, local minima

7) $\max = 11$ at $x = -3$, $\min = -14$ at $x = -2$

8) $\max = \sqrt{2}$ at $x = \frac{\pi}{4}$, $\min = -\sqrt{2}$ at $x = \frac{5\pi}{4}$