Section 2.1 Introduction to Limits

# Topic 1: Limits: A Graphical Approach

We write  and call *K* the **limit from the left** or the **left-hand limit** if  is close to *K* whenever *x* is close to, but to the left of, *c* on the real number line. When written , we read it as “*x* approaches *c* from the left” and means  and .

We write and call *L* the **limit from the right** or the **right-hand limit** if  is close to *L* whenever *x* is close to, but to the right of, *c* on the real number line. When written , we read it as “*x* approaches *c* from the right” and means  and .

**Theorem:** **On the Existence of a Limit**

For a two-sided limit to exist, the limit from the left and the limit for the right must exist and be equal. That is,

 if and only if .

# Topic 2: Limits: An Algebraic Approach

Let *f* and *g* be two functions, and assume that  and where *L* and *M* are real numbers (both limits exist). Then the following **limit properties** are true:

1.  for any constant *k*
2. 
3. 
4. 
5. 
6. 
7.  if 
8.  for  when *n* is even

**Theorem: Limits of Polynomial and Rational Functions**

1.  for any polynomial function *f*.
2.  for any rational function *r* with nonzero denominator at .

If  and , then  is said to be **indeterminate**, or more specifically, a  **indeterminate form**.

**Theorem: Limit of a Quotient**

If ,  and , then  does not exist.

# Topic 3: Limits of Difference Quotients

Let the function $f$ be defined in an open interval containing the number $a. $ One of the most important limits in calculus is the limit of the **difference quotient.**

