Abstracts of 2010 LSU Math REU talks

- **Name:** Suzanne Carter
  Title: Calculating Picard Fuchs Differential Equations
  Abstract: Families of algebraic varieties have an associated Picard Fuchs differential equation, with a solution space spanned by the periods of the variety. These periods are integrals of differentials taken over chains in the primitive cohomology of the varieties. The, often unique, holomorphic solution to this differential equation has interesting properties when looking at the coefficients; they satisfy congruences modulo powers of primes. We will discuss two methods for calculating these differential equations, a method outlined by Griffiths and a method that works for families of elliptic curves. Using the results of these techniques, we will discuss properties of these differential equations and the congruences that their solutions satisfy in certain examples.

- **Name:** Steffen Docken
  Title: Symmetric Square Differential Equations of the Beauville Families of Elliptic Curves
  Abstract: Each of the Beauville Families of elliptic curves has a corresponding differential equation called a Picard Fuchs differential equation. These Picard Fuchs differential equations are satisfied by modular forms. This talk will illustrate how the Symmetric Square differential equation for a family of elliptic curves can be derived from the Picard Fuchs differential equation. It will also show that the Symmetric Square differential equation is also satisfied by modular forms and that the coefficients of the solutions have interesting properties.

- **Name:** Sarah Loeb
  Title: Permissible Plane Embeddings of Dessin Blow-ups.
  Abstract: Given a ribbon graph embedded on a surface, we consider the blow-up, that is the three-valent partially oriented graph constructed by replacing each vertex by an oriented circle and attaching the edges around the circle according to the rotation system. A characterization in terms of two forbidden configurations is then given as to which blow-ups have permissible planar embeddings, i.e. those with embeddings where the orientation of the circles is determined by their nesting level. The proof requires Kuratowski's Theorem as well as an argument by cases based on connectivity. An application of this result is a characterization of which ribbon graphs arise as a state smoothing of a link diagram.

- **Name:** Natasha Potashnik
  Title:
  Abstract: Dessins d'enfants provide a surprising connection between group theory and topology. The absolute Galois group acts on dessins, and the orbits of this action are the main subject of our study. We will discuss invariants of these Galois orbits and methods for investigating them. In particular, we will explain the notion of a Belyi-extending function and demonstrate their utility in distinguishing Galois orbits. The second Chebyshev polynomial is known

to be an important Belyi-extending function. We conjecture that the nth Chebyshev polynomial T_n will in general yield no new 'useful' information. We show that if $\mathbb{D}$ is not a polygon, then $\mathrm{Aut}(T_n\mathbb{D}) = \mathrm{Aut}(\mathbb{D})$ for n positive odd and $\mathrm{Aut}(T_n\mathbb{D}) = \mathrm{Aut}(T_2\mathbb{D})$ for n positive even. We propose that a similar result holds for monodromy groups.

- **Name:** Shaunak Das
  Title: Differential Equations Satisfied by Families of Elliptic Curves
  Abstract: For a given family of elliptic curves, we are interested in the Picard-Fuchs differential equation which its periods satisfy; in particular, we wish to study the solution which is analytic at regular singularity at the origin. The requirement of analyticity guarantees a series expansion for our solution, whose coefficients have interesting arithmetic properties. In this paper, we first illustrate an example of an Atkin-Swinnerton-Dyer congruence satisfied by a number of Beauville's families of elliptic curves. We then present a family which does not satisfy this same special relation, and determine some new congruences which do in fact hold.

- **Name:** Laura G. Luttmer
  Title: Using Dessins d'Enfants to Understand the Absolute Galois Group
  Abstract: We examine the absolute Galois group over $\mathbb{Q}$ by studying the action of the Galois group on dessins d'enfants of genus 0. Special trees of small degree are drawn and their Belyi polynomials computed. We identify any rotational symmetries and non-trivial automorphism groups of the trees and compare the total number of trees produced to the number of trees calculated by the Frobenius formula. The order of the monodromy groups of the trees are calculated to identify which of the trees are special. This method is repeated to study the action of $PSL_2(\mathbb{F}_p)$ on $\mathbb{P}^1(\mathbb{F}_p)$. We examine the Belyi functions and dessin corresponding to the constellation

$$\left[x = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, y = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}, \text{ and } (xy)^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \right]$$

  when the genus is 0. The explicit relations of the functions with the canonical modular polynomials for $X_0(p)$ are found. These Belyi functions give a 1-parameter family of polynomials with Galois group $PSL_2(\mathbb{F}_p)$ over $\mathbb{Q}(\sqrt{p})$.

- **Name:** TEra Hartfield Title: Properties of Fibonacci Periods Abstract. In this work we study the properties of the functions $Q(m) = \frac{k(m)}{m}$, where $k(m)$ is the Fibonacci period mod m. Guo-Koch conjectured in 2009 that the set $\{m : Q(m) < 1\}$ is multiplicative. We found counterexamples to this conjecture numerically using a generalization of Walls Theorem. Then we produced an infinite family of counterexamples. We conjecture that the smaller set $\{m : Q(m) < 1, (m, 5) = 1\}$ is multiplicative.

- **Name:** Jordan Keller
  Title: Recursively Constructible Families of Dessin
  Abstract: An infinite sequence of dessin $\{\mathbb{D}_n\}$ is called recursive if the Bollobás-Riórdan-Whitney-Tutte (BRWT) polynomials $R(\mathbb{D}_n; x, y, z)$ satisfy a

linear recurrence relation whose coefficients are polynomials in $\mathbb{Z}[x, y, z]$. In this paper we present a method based on transfer matrices which shows recursiveness for a broad class of dessin sequences. Our method enables explicit, simplified computation of the BRWT polynomial for complex dessin in these sequences, as shown through several examples.

- **Name:** Kirby Fears
  Title: Dessins d'enfants: Symmetry and Quotient Dessins
  Abstract: This paper examines dessins d'enfants that adhere to symmetries defined by group actions. The quotient of a symmetric dessin is defined in order to describe a symmetric dessin in terms of a simpler figure (the quotient) from which the whole dessin can be constructed. This concept is analogous to analyzing links in terms of quotient links. Tools include the Skein relations for dessins and the Bollobás-Riordan-Whitney-Tutte (BRWT) polynomial. Relations between dessins and their quotients are primarily in terms of relations between the BRWT of a dessin compared to that of its quotient. The most fundamental comparison is whether a dessin's BRWT is divisible by its quotient's BRWT.

- **Name:** Daniel C. Thompson
  Title: Relations in the Grothendieck Ring of Dessins
  Abstract: The author introduces multigraphs and dessins (graphs embedded in orientable surfaces) and provides a concise summary of W. H. Tutte's ring of graphs under the contraction-deletion relation. The analogous ring of dessins is considered, and a small class of one-vertex dessins are shown to generate the ring. The problem of finding a change-of basis in the ring $\mathbb{Z}[x_{i,j}]$ to show independence is discussed. A discussion of the Tutte polynomial and the Bollobás–Riordan–Tutte polynomial as V–functions is included.

- **Name:** Daniel Sieburth
  Title: The p-adic Convergence of the Ihara Zeta Function on Families of Graphs
  Abstract: We generalize Munch's proof of the 3-adic convergence of the Ihara Zeta Function on the Sierpinski's gasket to include all families of graphs that satisfy certain criteria relating to symmetry and self-similarity. We then weaken these conditions to include only the self-similarity condition. Finally, we rephrase these self-similarity conditions with a *patching* approach, and examine some other properties of the Ihara Zeta Functions within some of these families of graphs.

- **Name:** Katherine Raoux
  Title: Belyi Extending Maps and Dessin d'Enfant
  Abstract: Little is currently known about the structure of the absolute Galois group over the rational numbers. In 1979, Belyi proved that a Riemann surface X is associated with algebraic curve defined over $\bar{\mathbb{Q}}$ if and only if it has a Belyi function. Thus, the absolute Galois group acts on Belyi maps. Grothendieck intrduced dessin denfants as a way to study, combinatorially, the action of the absolute Galois Group on Belyi functions. He also noticed

that the absolute Galois group acts faithfully on dessin denfants . We study this action by looking at invariants of the Galois orbit of dessin including the automorphism and monodromy groups. In addition we define Belyi-extending maps and summarize Melanie Woods geometric and combinatorial methods for extending dessin. We reexamine Melanie Woods example of a new invariant that distinguishes two previously indistinguishable dessin and show the effect of the Belyi extension on the automorphism group of the dessin.

Neal W. Stoltzfus