Section 4.1 First Derivative and Graphs

# Topic 1: Increasing and Decreasing Functions

**Theorem: Increasing and Decreaing Functions**

For the interval , if , then *f* is increasing, and if  , then *f* is decreasing.



The intervals on which a function *f* is increasing and decreasing must always be expressed in terms of open intervals that are subsets of the domain of *f.*

# Topic 2: Local Extrema

A real number *x* in the domain of *f* such that  or  does not exist is called a **critical number** of *f*.

Critical numbers of *f* belong to the domain of *f* and are partition numbers for . But  may have partition numbers that do not belong to the domain of *f* and therefore are not critical numbers of *f.*

Do not assume that all partition numbers for the derivative of *f* are critical numbers of the function *f*. To be a critical number of *f*, a partition number for must also be in the domain of *f.*

In general, we call  a local maximum if there exists an interval  such that for all *x* in *.* The value  is called a local minimum if there exists an interval such that for all *x* in. The value  is called a **local extrema** if it is either a local maximum or a local minimum.A point on a graph where a local extremum occurs is also called a **turning point.**

**Theorem: Local Extrema and Critical Numbers**

If  is a local extrema of the function *f,* then *c* is a critical number of *f*.

# Topic 3: The First Derivative Test

**First Derivative Test for Local Extrema**

Let *c* be a critical number of *f* where  is defined and either  or  is not defined. Construct a sign chart for  close to and on either side of *c.*

| **Sign chart** |  |
| --- | --- |
| f prime of x is negative just to the left of c and changes to positive at c, so f is decreasing to the left of c and increasing to the right. |  is a local minimum.If  changes from negative to positive at *c*$,$ then  is a local minimum.  |
| f prime of x is positive just to the left of c and changes to negative at c, so f is increasing to the left of c and decreasing to the right. |  is a local maximum.If  changes from positive to negative at *c*, then  is a local maximum. |
| f prime of x is positive on both sides of c, so f is increasing on both sides of c. |  is not a local extremum.If  does not change signs at *c*, then  is neither a local minimum nor a local maximum. |
| f prime of x is negative on both sides of c, so f is decreasing on both sides of c. |  is not a local extremum.If  does not change signs at *c*, then  is neither a local minimum nor a local maximum. |

**Theorem: Intercepts and Local Extrema of Polynomial Functions**

If , , is a polynomial function of degree  , then *f* has at most *n x*-intercepts and at most  extrema.

# Topic 4: Economics Applications