Section 4.5 Absolute Maxima and Minima

# Topic 1: Absolute Maxima and Minima

One of the most important applications of the derivative is to find the absolute maximum or minimum value of a function. An economist may be interested in the price or production level of a commodity that will bring a maximum profit. A doctor may be interested in the time it takes for a drug to reach its maximum concentration in the bloodstream after an injection.

Recall that  is a local maximum if  for *x* near *c* and a local minimum if  for *x* near *c.*

If  for all *x* in the domain of *f*, then  is called the **absolute maximum** of *f.* If for all *x* in the domain of *f*, then  is called the **absolute minimum** of *f.* An absolute maximum or absolute minimum is called an **absolute extremum**.

**Extreme Value Theorem**

A function *f* that is continuous on a closed interval  has both an absolute maximum and absolute minimum on that interval.

**Theorem: Absolute Extrema**

Absolute extrema (if they exist) must always occur at critical values or at endpoints.

**Procedure: Finding absolute extrema on closed intervals**

1. Check to make certain that *f* is continuous over .
2. Find the critical numbers in the interval .
3. Evaluate *f* at the endpoints *a* and *b* and at the critical numbers found in step 2.
4. The absolute maximum of *f* on is the largest value found in step 3.
5. The absolute minimum of *f* on is the smallest value found in step 3.

# Topic 2: Second Derivative and Extrema

**Second Derivative Test for Local Extrema**

Let *c* be critical number of  such that . If , then  is a local minimum. If , then  is a local maximum.

|  |  | **Graph of *f* is:** |  | **Example** |
| --- | --- | --- | --- | --- |
| 0 | positive | concave upward | Local minimum | a u-shaped curve |
| 0 | negative | concave downward | Local maximum | an upside down u-shaped curve |
| 0 | $$0$$ | ? | Test does not apply |  |