Three Dimensional Modeling of Various Topological Surfaces

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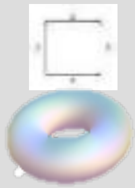
Introduction: The focus of this course was on the methods and strategies for distinguishing between surfaces. Different surfaces were introduced in class, then the surfaces were modeled using the computer algebra system, Mathematica. Finally the Mathematica files were converted into STL (triangulation) files to be 3D printed.

Surface Classifications

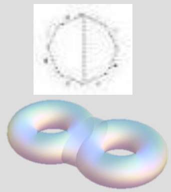
Theorem: Every compact connected surface is homeomorphic to a sphere, a connected sum of tori or a connected sum of projective planes.

Tools are needed to understand surfaces as the sum of their components. One such tool is the planar diagram. Planar diagrams are polygons with pairs of oriented edges. The paired edges are identified together according to their orientation to represent a surface. Below are some surfaces represented by their planar diagrams and their 3D plots.



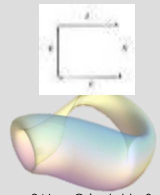
Genus 1 Orientable Surface (Torus)

T#T



Genus 2 Orientable Surface (Double Torus)

RP2#RP2



Genus 2 Non-Orientable Surface (Klein Bottle)

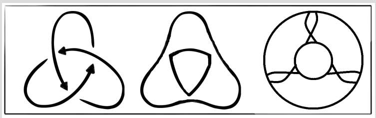
Knots and Seifert

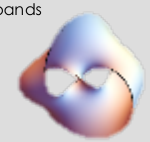
Theorem: Every knot is the boundary of an orientable surface

A Seifert Surface is a two-dimensional, orientable surface whose boundary is a knot. Seifert Surfaces are created to more easily understand properties of the knots from which they arise. They also serve as an aid in distinguishing between different knots

Seifert's algorithm:

1. Assign an orientation to the planar diagram of a knot
2. Eliminate all crossings, forming oriented disks
    1. If a disk is nested inside another, offset it perpendicular to the plane in which it lies
3. For each eliminated crossing, connect the disks with twisted bands





Projections

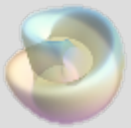
A projection is the transformation of points from one space or plane onto another by associating corresponding points from one space to the other. One type of projection is of a 4-dimensional object into 3 dimensions. This can be done by using different 3-combinations of the four dimensions. This is shown below with the 4D Klein bottle.

$$x=\cos(\theta)(c+\cos(\tfrac{\theta}{2})\sin(\varphi)-\sin(\tfrac{\theta}{2})\sin(2\varphi))$$

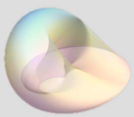
$$z=\sin(\tfrac{\theta}{2})\sin(\varphi)+\cos(\tfrac{\theta}{2})\sin(2\varphi)$$

$$y=\sin(\theta)(c+\cos(\tfrac{\theta}{2})\sin(\varphi)-\sin(\tfrac{\theta}{2})\sin(2\varphi))$$

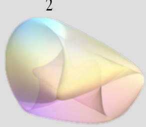
$$w=\sin(\tfrac{\theta}{2})\cos(\varphi)+\frac{(\sin(\tfrac{\theta}{2})\cos(2\varphi))}{2}$$



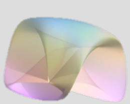
(x,y,z)



(x,y,w)



(x,z,w)

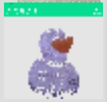


(y,z,w)

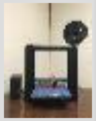
Technology

The Raspberry Pi is a low cost, credit-card sized fully functioning computer that plugs directly into a monitor, and uses a standard keyboard and mouse. The Raspberry Pi has several built in programs including Mathematica. Weekly projects were created on the Raspberry Pi to create surfaces to 3D print.

A 3D scanner scans an object and collects the data on its shape. It then converts the data to an STL file. An STL file describes a raw unstructured triangulated surface using a 3D Cartesian coordinate system. From the scan, the object can be manipulated on the computer and then sent to the 3D printer to be printed.



A 3D printer produces solid objects by adding continuous layers of material, from the bottom up, until the entire object is formed. The input is a STL file on a microSD card produced from a Mathematica file or a 3D scan. The output is a 3D object with support structures automatically included to print successfully; these supports are easily broken off.



Conclusion: The study of surfaces and topology is vast and detailed. There are many ways to analyze surfaces ranging from the use of only a pen and paper to precise 3D printing machines. While all the tools of analysis are useful, the higher technology modeling systems are preferred. They are simple, intuitive and allow rapid and accurate creation of computer and physical models.

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