Section 7.5 Linear Programming

**Linear programming** is a method for solving problems in which a particular quantity that must be maximized or minimized is limited or restricted by other factors.

Linear programming is one of the most widely used tools in management science. It helps businesses allocate resources to manufacture products in a way that will maximize profit. Profit can be modeled as a function.

An algebraic expression in two or more variables describing a quantity that must be maximized or minimized is called an **objective function**. In the example above, the profit equation would be the objective function.

A system of linear inequalities is used to represent restrictions, or **constraints**, on the function that must be maximized or minimized.

# Objective 1: Use graphs to determine the maximum and the minimum of an objective function

The maximum and minimum values of the objective function occur at one or more of the corner points of the graph of the system of constraints.

Thus, to find the maximum and minimum of the objective function that is subject to constraints, simply substitute the values of the corner points of the graphed region into the objective function. Then use those values to determine the maximum value or minimum value of the objective function.

# Objective 2: Use linear programming to solve application problems

The steps to solving a linear programming problem are summarized below.

**Solving a Linear Programming Problem**

Let $z=ax+by$ be an objective function that depends on *x* and *y*. Furthermore, *z* is subject to a number of constraints on *x* and *y*. If a maximum or minimum value of *z* exists, it can be determined as follows:

1. Graph the system of inequalities representing the constraints.
2. Find the value of the objective function at each corner, or vertex, of the graphed region. The maximum and minimum of the objective function occur at one or more of the corner points.