Section 6.5 Negative Exponents

# Objective 1: Evaluating Expressions Containing Negative Exponents

In section 6.1, we worked with whole number exponents. In this section, we will expand our work with exponents to include exponents that are negative integers.

Consider the expression $\frac{x^{2}}{x^{5}}$. Applying the quotient rule,

$\frac{x^{2}}{x^{5}}=x^{2-5}=x^{-3}$.

Applying the fundamental principle for fractions,

$\frac{x^{2}}{x^{5}}=\frac{x⋅x}{x⋅x⋅x⋅x⋅x}=\frac{1}{x^{3}}$.

Since $\frac{x^{2}}{x^{5}}=x^{-3}$ and $\frac{x^{2}}{x^{5}}=\frac{1}{x^{3}}$, then $x^{-3}$ must equal $\frac{1}{x^{3}}$.

**Negative Exponents**

If $a$ is a real number other than $0$ and $n$ is an integer, then

$a^{-n}=\frac{1}{a^{n}}$.

Evaluate the expression.

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| a. $6^{-2}$ | b. $\left(-3\right)^{-4}$ | c. $\left(\frac{1}{2}\right)^{-3}$ |

# Objective 2: Simplifying Exponential Expressions Containing Negative Exponents

All of the previously stated rules for exponents apply for negative exponents also. Here is a summary of the rules and definitions for exponents.

**Summary of Exponent Rules**

If $m$ and $n$ are integers and $a$, $b$, and $c$ are real numbers, then:

Product rule for exponents: $a^{m}⋅a^{n}=a^{m+n}$

Power rule for exponents: $\left(a^{m}\right)^{n}=a^{m⋅n}$

Power of a product: $\left(ab\right)^{n}=a^{n}⋅b^{n}$

Power of a quotient: $\left(\frac{a}{b}\right)^{n}=\frac{a^{n}}{b^{n}}$, $b\ne 0$

Quotient rule for exponents: $\frac{a^{m}}{a^{n}}=a^{m-n}$, $a\ne 0$

Zero exponent: $a^{0}=1$, $a\ne 0$

Negative exponent: $a^{-n}=\frac{1}{a^{n}}$, $a\ne 0$

Simplify the expression by rewriting using only positive exponents.

|  |  |
| --- | --- |
| a. $4a^{-9}$ | b. $\frac{3}{b^{-5}}$ |
| c. $\frac{x^{-8}}{x^{2}}$ | d. $\frac{15p^{6}q}{3p^{-1}q^{4}}$ |
| e. $\left(x^{2}y^{4}\right)^{-2}$ | f. $(2ab^{-3})(-4a^{-3}b^{2})$ |