OAL-14

Conference on Order, Algebra and Logic
Louisiana State University, Baton Rouge LA
May 1–3, 2014

ABSTRACTS of TALKS

The schedule of talks is posted at the conference web site:
`https://www.math.lsu.edu/OrderedAlgebraicStructures2014`

Richard N. Ball. University of Denver, Denver, Colorado 80210 USA. 40 minutes
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The Pointfree Representation of Truncated Archimedean Lattice Ordered Groups

In this talk we develop the analog for truncated archimedean ℓ-groups, hereafter referred to as truncs, of Madden's pointfree representation for $\mathbf{W}$, the category of archimedean ℓ-groups with designated weak order unit. In the first part of the talk we will motivate the notion of truncation with a few sketches, review its definition, and outline the pointed, or Yosida representation of truncs. We shall point out the necessity of using pointed spaces and maps in order to render the representation functorial. In the second part of the talk we will discuss the main result, which is the pointfree representation of truncs.

Theorem. *For every archimedean trunc A there is a regular Lindelöf pointed frame* L, *a subtrunc* $\widehat{A}$ *of* $\mathcal{R}_0\mathrm{L}$, *and a trunc isomorphism* $A \to \widehat{A}$. *The pointed frame* L *is unique with respect to its properties, and the representation is functorial.*

A pointed frame is just a (completely regular) frame L equipped with a designated point $* : \mathrm{L} \to 2$; for example, the pointed frame of the reals, designated $\mathcal{O}_0\mathbb{R}$, is just the topology $\mathcal{O}\mathbb{R}$ of the real numbers equipped with the frame map $\mathcal{O}\mathbb{R} \to 2$ corresponding to the constant 0 function $\mathbb{R} \to 0$. A pointed frame map is just a frame map which commutes with the designated points, and $\mathcal{R}_0\mathrm{L}$ stands for the trunc of pointed frame maps $\mathcal{O}_0\mathbb{R} \to \mathrm{L}$. We shall conclude the talk by pointing out that $\mathbf{W}$ is a non-full bireflective subcategory of the category of truncs.

Karim Boulabiar. Tunis-El Manar University, Tunisia. 30 minutes
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Extreme contractive operators on Stone f-algebras

Let A be an archimedean semiprime (i.e., reduced) f-algebra and $\mathrm{Orth}(A)$ denote the archimedean f-algebra of all orthomorphisms (i.e., bounded polar preserving linear maps) on A. The identity map I_A of A is the unit element in $\mathrm{Orth}(A)$ and A can be considered as a a ring ideal and a sublattice of $\mathrm{Orth}(A)$. We call A a *Stone f-algebra* if

$$f \wedge I_A \in A \quad \text{for all } f \in A.$$

Let A, B be two Stone f-algebras. A linear map $T : A \to B$ is said to be *contractive* if

$$f \in A \text{ and } 0 \leq f \leq I_A \text{ imply } 0 \leq Tf \leq I_B.$$

The set $\mathcal{K}(A, B)$ of all contractive linear maps from A into B is a convex set. That is,

$$\lambda \in [0, 1] \text{ and } S, T \in \mathcal{K}(A, B) \text{ imply } \lambda S + (1 - \lambda)T \in \mathcal{K}(A, B).$$

We prove that if $T \in \mathcal{K}(A, B)$, then the following are equivalent.

(*i*) T is an extreme point in $\mathcal{K}(A, B)$.

(*ii*) T is a ring homomorphism.

(*iii*) T is a *Stone operator*, that is, the equality $T(I_A \wedge f) = I_B \wedge Tf$ holds in B for all $f \in A$.

This extends earlier results by Ellis, Ionesco Tulcea, and Bonsall-Lindenstrauss-Phelps for $C(X)$-type algebras and those of Hager-Robertson, van Putten, and Huijsmans & de Pagter for unital f-algebras. This talk is based upon a joint work with Ben Amor and El Adeb.

Gerard Buskes. University of Mississippi, University, Mississippi 38677 USA. 30 minutes
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Polynomials on Vector Lattices and Banach Lattices

I will present results on the lattice structure of homogeneous polynomials on vector lattices and Banach lattices. The results were obtained in joint work with. Q. Bu, A. Popov, V. Troitsky, and A. Tcaciuc.

Charles N. Delzell . Louisiana State Univ., Baton Rouge LA 70803 USA. 30 minutes
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A New, Simpler, Finitary Construction of the Real Closure of a Computable Ordered Field

We give a new, simple, finitary construction of the real closure R of a computable ordered field $(K, \geq)$, as the set of equivalence classes of uniquely satisfiable formulae with one free variable in the first-order language of ordered rings $(+, -, \cdot, 0, 1, \geq)$ with equality, augmented by a constant symbol c_k for each element $k \in K$. It is routine to verify, finitarily, that this R satisfies the axioms of real closed, ordered fields, with the exception of the axiom $0 \neq 1$, for which the verification is difficult, and depends on (and is equivalent to) a finitary proof of the consistency of the theory of real closed ordered fields augmented by the (atomic) diagram of $(K, \geq)$.

Benjamin Forrest Dribus. Louisiana State Univ., Baton Rouge LA 70803 USA. 30 minutes
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Path Algebras in Quantum Causal Theory

A directed path in an acyclic directed graph G is an order morphism from a discrete linearly ordered set into the edge set of G. A path algebra over G is an algebra generated by directed paths in G, with multiplication given by concatenation. In physics, order often signifies information flow or causal influence. In particular, directed paths underlie the path integral approach to quantum theory, which leads to Schrödingers equation. This talk describes how path algebras may be used to derive analogues of Schrödingers equation in the discrete order-theoretic setting.

Themba Dube*. University of South Africa (Unisa), South Africa. 20 minutes
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Coauthor: **Tega Ighedo**. University of South Africa (Unisa), South Africa.

Another View of OZ

A frame L is called an*Oz-frame* if, for every $a \in L$, the open quotient map $L \to \downarrow a$ is coz-onto. This is equivalent to saying every regular element of L is a cozero element. In this talk we will give a ring-theoretic characterization of these frames. We will also report on necessary and sufficient conditions that βL be Oz.

Anthony W. Hager. Wesleyan University, Middletown, Connecticut 06459 USA. 40 minutes
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Some Unusual Epicomplete Archimedean Lattice-Ordered Groups

An Archimedean ℓ-group is *epicomplete* if it is divisible and σ-complete, both laterally and conditionally. Under various circumstances it has been shown that epicompleteness implies the existence of a compatible reduced f-ring multiplication; the question has arisen whether or not this is always true. We show that a set-theoretic condition weaker than the continuum hypothesis implies "not", and conjecture the converse. The examples also fail decent representation and existence of some other compatible operations.

W. Charles Holland* University of Colorado, Boulder, Colorado 80309 USA. 20 minutes
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Very Generalized Commutativity of Lattice-Ordered Groups

Generalized commutativity of lattice-ordered groups has been going on for quite a while. We are going to define some extremely generalized versions of lattice ordered groups. And we will prove which ones of these are actually varieties.

Oghenetega Ighedo. University of South Africa (Unisa), South Africa. 20 minutes
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On the Projectability Properties of Frames of z-Ideals and d-Ideals of $\mathcal{R}L$

Let $\mathrm{Rad}(\mathcal{R}L)$ denote the frame of radical ideals of the ring $\mathcal{R}L$ of continuous real-valued functions on a completely regular frame L. The lattices $\mathrm{Zid}(\mathcal{R}L)$ and $\mathrm{Did}(\mathcal{R}L)$ of z-ideals and d-ideals of $\mathcal{R}L$, respectively, are quotients of $\mathrm{Rad}(\mathcal{R}L)$, and are therefore frames. In this talk I will report on the projectability properties of these frames.

Jingjing Ma. University of Houston-Clear Lake, Houston, Texas 77058 USA. 40 minutes
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Positive Derivation on Lattice-Ordered Rings—Old and New Results

Let R be a lattice-ordered ring. A derivation d on R is called *positive* if $\forall x \geq \exists 0$, $d(x) \geq \exists 0$. Positive derivations on lattice-ordered rings were first studied by P. Colville, G. Davis and K. Keimel for f-rings. They proved that for an Archimedean f-ring R and a positive derivation d on R, $d(R) \subseteq \ell - N(R)$ and $d(R^2) = \{0\}$, where $\ell - N(R)$ is the ℓ-radical of R. In this talk I will present some old results and new development on positive derivations of lattice-ordered rings since then. (This presentation is supported by NSF, Grant No. 1060039.)

James Madden. Louisiana State University, Baton Rouge LA 70803 USA. 30 minutes
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Neighbors in Subsets of $\mathbb{Z}^n$

Let N be a finite subset of $\mathbb{Z}^n$ with coordinate-wise supremum $\vee N$. The *interior of the shadow of* N is the set of elements of $\mathbb{Z}^n$ that are strictly less than $\vee N$ in every coordinate. If A is an antichain in $\mathbb{Z}^n$, then a finite subset N of A is said to be *neighborly in* A if the interior of the shadow of N contains no elements of A. This concept has a remarkable range of applications in integer programming, semigroup theory and combinatorial commutative algebra. I will illustrate.

Warren Wm. McGovern*. Florida Atlantic Univ., Boca Raton, FL 33431 USA. 40 minutes
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Coauthor: **Jingjing Ma**. University of Houston-Clear Lake, Houston, Texas 77058 USA.

Regular Division Closed ℓ-Rings

$(R, \leq)$ denotes a commutative *ell*-ring (not necessarily unital) containing a positive regular element. Our investigations center around the question of when one can order $q(R)$, the classical ring of quotients of R, in such a way to ensure that R is ℓ-embedded in $q(R)$ and so that the inverse of a positive element of R is positive in $q(R)$. This leads to the investigation of what we call regular division closed ℓ-rings as well as the construction of a new type of positive element which we call a separating element; the notion is weaker than a d-element.

Trevor McGuire. Louisiana State University, Baton Rouge LA 70803 USA. 30 minutes
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Combinatorial Minimal Free Resolutions

For a given lattice $\Lambda \subset \mathbb{Z}^n$, define a binomial ideal in $k[x_1, \ldots, x_n]$ by $I_\Lambda =< X^{\lambda^+} - X^{\lambda^-} \mid \lambda \in \Lambda >$. Similarly, for a finite set $A \subset \mathbb{N}^n$, define a monomial ideal by $I_A =< X^\alpha \mid \alpha \in A >$. Combinatorial algorithms that generate (minimal, free) resolutions of both of these ideals have been previously discovered. In this talk, we will discuss a new combinatorial algorithm that generates a free resolution of the ideal $I = I_\Lambda + I_A$ under mild conditions on Λ and A. In $k[x_1, x_2, x_3]$, the algorithm generates a minimal free resolution. More generally, if we have a lattice $\Lambda \subset \mathbb{Z}^n$, and a set $A \subset \mathbb{Z}^n$ such that $A = A_0 + \Lambda$ for some $A_0 \subset A$, then under similar mild conditions, our algorithm generates a minimal free resolution of $I_A = I_\Lambda + I_{A_0}$. As an intermediate step, we will resolve the ideal $(I_{A_0} + I_\Lambda)/I_\Lambda \subset k[x_1, \ldots, x_n]/I_\Lambda$, which is a monomial ideal generated by monomials of the form $X^\alpha + I_\Lambda$ for $\alpha \in A_0$. Computationally supported conjectures will be discussed if time permits.

Inderasan Naidoo*. University of South Africa (Unisa), South Africa. 20 minutes
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Coauthor: **Themba Dube**. University of South Africa (Unisa), South Africa.
Coauthor: **Charles Msipha**, Tshwane University of Technology, South Africa.

On Almost Realcompact Frames*

We characterize the topological notion of almost* realcompactness for the category of frames. We also look at preservation of this property across particular frame homomorphisms, and consequences of almost* realcompactness with zero-dimensionality.

Philip Scowcroft, Wesleyan University, Middletown, Connecticut 06459, USA. 40 minutes
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Existential Closures of Abelian Lattice-Ordered Groups

Let G be an Abelian lattice-ordered group ("ℓ-group," in what follows). G is said to be *existentially closed* just in case any finite system of ℓ-group equations and inequations over G that is solvable in some ℓ-group extending G is already solvable in G. If H is an ℓ-group, an ℓ-group G extending H is called an *existential closure of* H just in case G is existentially closed and embeds over H into any existentially closed ℓ-group extending H. This talk will discuss the extent to which ℓ-groups H have existential closures unique up to isomorphism over H.

Piotr J. Wojciechowski, Univ. of Texas at El Paso, El Paso, TX 79968. 30 minutes
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Multiplicative Decomposition Property for Matrix Algebras and its Consequences

We consider directly ordered real algebras of matrices with the following Multiplicative Decomposition Property: if $A, B \geq \mathbf{0}$ and $\mathbf{0} \leq C \leq AB$, then there exist $A', B' \geq \mathbf{0}$ such that $A' \leq A$, $B' \leq B$ and $C = A'B'$. We have obtained necessary and sufficient conditions characterizing them. These algebras are lattice-ordered. A well-known algorithm for checking the lattice property of a subspace was reduced in complexity from exponential to polynomial. Another property lead to some combinatorial considerations. In particular, there exists an isomorphism-preserving one-to-one correspondence between a subclass of these algebras and the class of all bipartite graphs.

Brian Wynne, Bard College at Simon's Rock, Great Barrington, MA 01230 USA. 20 minutes
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Upper Products of Existentially Closed Abelian ℓ-Groups (Redux)

Given abelian ℓ-groups A and B and a lattice homomorphism π from the principal convex ℓ-subgroups of A into the summands of B with $\pi(0) = 0$, the upper product construction of Ball, Conrad, and Darnel provides an ℓ-group structure on the cartesian product $A \times B$ in which "A lies above B." At OAL13, I presented conditions under which such an upper product of two existentially closed abelian ℓ-groups is existentially closed, but was unable to give an example in which those conditions were satisfied. In this talk I will supply such an example.

Taewon Yang, Florida Atlantic University, Boca Raton, Florida 33431 USA. 30 minutes
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f-Rings $\mathcal{Z}L$

We introduce the equational class of unital commutative f-rings that are modeled after the classical ring $C(X, \mathbb{Z})$ of all integer-valued continuous maps on a topological space X. We then show that if a ring A in this class is bounded, meaning $|a| \leq n$ for some $n \in \mathbb{N}$ (for all $a \in A$), then A indeed arises from the f-ring $\mathcal{Z}L$ of all bounded integer-valued continuous maps on some compact zero-dimensional σ-frame L. We also note that the process involved in such a representation can be used to obtain the compact zero-dimensional coreflections from arbitrary σ-frames.