Analytic Number Theory;
An Introduction

Instructor: William James Martin, University of New Orleans

This course will be a self contained introduction to number theory utilizing techniques of analysis.

Topics to be studied will be the Fundamantal theorem of arithmetic, the Euclidean algorithm, prime numbers, the arithmetical functions, including the Euler totient function, the Mobius, the Bell, the Mangoldt functions, Dirichlet multiplications, the average orders of the arithmetical functions, some elementary theorems on the distribution of primes, congruences, and, if time permits, Gauss's quadratic reciprocity law.

Techniques of complex analysis will be utilized but will be introduced and explained as needed. There is no prerequisite beyond three semesters of calculus.

Projects: (a) The Gamma function

(b) the Reimann Zeta function

(c) Sieve theory

Testbooks: We will mainly follow *Analytic Number Theory,* by Tom Apostol, though the lectures should be self contained, so that purchasing the text book would not be essential. Other helpful texts are, (i), *Introduction to Number Theory*, by Hardy and Wright, *Riemann's Zeta Function,* by Harold Edwards, *The Prime Numbers,* by Gerald Tannenbaum, *Theory of Functions, parts I and II,* by Konrad knopp, *Functions of a Complex Variable*, by John Conway, *Gamma,* by Julian Havil. The latter is a very readable book which the student would no doubt enjoy independently of the course. (All of these books are available on amazon.com.)

Instructor: Professor Vlajko Kocic, Xavier University of Louisiana

Mathematical Modeling in Life Sciences: Introduction to mathematical models and techniques in life sciences including topics in population biology and epidemiology, cell division, bacterial growth in a chemostat, host-parasitoid systems, and predator-prey systems. The mathematical topics include linear and nonlinear difference equations, in particular the logistic equation, continuous processes described by ordinary differential equations, stability considerations including chaos (for both discrete and continuous models). Use of the technology is integrated in the course.

Projects.

1. Study the dynamics of the discrete Sigmoid Beverton-Holt population model:

$$x_{n+1} = \frac{a x_n^{\delta}}{1 + x_n^{\delta}}, n = 0, 1, \ldots$$

   where $a, \delta > 0$ and initial condition $x_0 > 0$. The model is a generalization of well-known Beverton-Holt model ($\delta = 1$) and it exhibits so-called "Allee effect" (population is heading to extinction at low population densities). The existence of equilibria, their stability, the existence of periodic solutions, and chaotic behavior will be studied. The approach includes both theoretical considerations and computer simulations. In addition, more advanced work is related to the study of the periodically forced model, when the parameter a is replaced with the positive periodic sequence $\{a_n\}$ with prime period p:

$$x_{n+1} = \frac{a_n x_n^{\delta}}{1 + x_n^{\delta}}, n = 0, 1, \ldots$$

2. Study the dynamics of the generalized Ricker's population model:

$$x_{n+1} = x_n^{\delta} e^{r - x_n}, n = 0, 1, \ldots$$

   where $r, \delta > 0$ and initial condition $x_0 > 0$. The above model is a generalization of well-known Ricker's model ($\delta = 1$) and it exhibits so-called "Allee effect" (population is heading to extinction at low population densities). Ricker's equation is well-know and it exhibits complex dynamics including chaos for some values of r. Existence of equilibria, their stability, existence of periodic solutions, and chaotic behavior will be studied. The approach includes both theoretical considerations and computer simulations. In addition, more advanced work will include the study of the generalized Ricker's equation with the delay,

$$x_{n+1} = x_n^{\delta} e^{r - x_{n-k}}, n = 0, 1, \ldots$$

   where k is a positive integer. The focus is on the case when $k = 1$ and the local stability, boundedness, and the existence of unbounded solutions will be the main focus of study. This project may lead to some original results.

3. Study the dynamics of the discontinuous piecewise linear map of the form

$$f(x) = \begin{cases} ax & if \quad x \in [0, T) \\ bx + c & if \quad x \in [T, \infty) \end{cases}$$

where $a, b, T > 0, c \in$ □ . The difference equations of the form

$$x_{n+1} = f(x_n), n = 0, 1, \ldots$$

where f is piecewise linear map defined above are used in literature, for example in population dynamics and in the study of models of spread of epidemics. Some special cases have fascinating properties. The study could lead to some original results. Main tools will include computer experimentation and some theoretical work.

Textbook: Instructors notes and handouts

(recommended, not required) L. Edelstein-Keshet, Mathematical Models in Biology, *SIAM Publishers,* 2005

E. S. Allman, J. A. Rhodes, Mathematical Models in Biology, an Introduction, *Cambridge University Press*

Software E & F Chaos (software), and the documentation could be downloaded from http://www1.fee.uva.nl/cendef/whoiswho/makehp/page.asp?iID=19

C. Diks, C. Hommes, V. Panchenko, R. vander Wielde, E&F Chaos: a user friendly software package for nonlinear dynamics, *Computational Economics* 32, 221 – 244.

C. Diks, C. Hommes, V. Panchenko, M. Tyszler, *Manual E&F Chaos Program.*

Symmetry and Groups

The notion of symmetry is seen everywhere. Examples are found in the human face, and in the repeating patterns found in wallpaper. Mathematically, symmetry is captured by the concept of a *group*. In this course, groups will be introduced and lots of explicit examples will be studied. No familiarity with groups will be assumed.

It is assumed that the student

- has studied two or three semesters of calculus,
- knows a bit about 2-by-2 matrices, including determinants, and
- is eager to learn new things, and
- is willing to work hard and not get discouraged.

While there is no required book for this course, it is *strongly recommended* that the student purchase the two books below, as references.

1. Combinatorics Including Concepts of Graph Theory, V. K. Balakrishnan, Schaum's Outlines, McGraw-Hill, 1995. This is available in paperback from Amazon for $14.78.
2. Groups and Symmetry, a Guide to Discovering Mathematics, David W. Farmer, Mathematical World, vol. 5, American Mathematical Society, 1996. This is available in paperback from Amazon for $19.80 .

These books are for you to read and consult, and think about. We will not follow either text, but nearly all the material we will discuss is found in one or the other of these books.

Projects:

1. The 15-puzzle is a popular game in which 15 square tiles can slide inside a 4-by-4 frame. The project is to describe the symmetry group of the puzzle. In particular, starting from the standard position with the tiles in order from 1 to 15 (going left to right and top to bottom, ending with an empty space), can one end up with the tiles in reverse order from 15 to 1 (going left to right and top to bottom, ending with an empty space) using only legal moves?

2. A *strip pattern* is a special type of 2-dimensional pattern. A symmetry of a strip patter is a rigid motion of the plane that takes the pattern back to itself. Find all possible groups of symmetries of a strip pattern. For each group, give one strip pattern having that group of symmetries. Finally, use the groups to classify the strip patterns.

Fourier Analysis and Applications

Instructor: Mark G. Davidson

The simplest kinds of functions we encounter early on in mathematics are the polynomials, i.e. linear combinations of powers of x. In calculus we learn that more complicated functions can sometimes be represented as a power series, a sort of infinite polynomial. For example

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

More generally, the representation of functions into simpler terms is a fundamental problem in mathematics. Fourier Analysis grew out of representing functions as series of terms involving $\sin nx$ and $\cos nx$ (instead of powers of x) called Fourier Series.

Today Fourier analysis has many scientific applications in physics, partial differential equations, number theory, combinatorics, signal processing, imaging, probability theory, statistics, option pricing, cryptography, numerical analysis, acoustics, oceanography, optics, diffraction, geometry, and other areas. This course will begin with the basics of Fourier Series and as time permits we will explore some of the applications mentioned above. It is assumed that the student has taken two semesters of calculus. It would be helpful, but not necessary, to have taken a course in differential equations.

I will cover topics found in the following **required** book:

'Fourier Series', by Georgi. P. Tolstov. I found this book on Amazon for $11.53.

I also **recommend** the book:

'Fourier Analysis with applications to Boundary Value Problems' by Murray Spiegel in the Schaum's Outline Series. I found this book on Amazon for $13.57.

Project 1: **Bessel Functions and Applications**

You will start with the definition of the series of Bessel function and then explore their orthogonality relations and other functional relations. Possible areas of application include the solution to the heat equation.

Project 2 **Orthogonal Systems of Special Functions**

Start with the definition of an orthogonal system. Examples include the Hermite, Laguerre, and Legendre functions. Consider their generating functions and applications.