

Math Tune up Summer 2008

Exercises

Wed. August 13

1. Solve the following equations

(a)

$$x \frac{dy}{dx} - 3y = x^4$$

(b)

$$y' + y = 2xe^{-x} + x^2$$

2. Check that $y = x^3$ is a solution of

$$y' = \frac{y^2}{x^4} + 2x^2$$

then solve this equation.

3. Remember that an equation of the form

$$y' + P(x)y = Q(x)y^n$$

is called a *Bernoulli's* equation. It can be reduced to a linear equation by the change of variable $z = y^{1-n}$. Solve:

(a)

$$xy' + y = x^4y^3$$

(b)

$$xy^2y' + y^3 = x \cos x$$

4. Solve the following differential equation and determine the interval of validity for the solution

$$\frac{dy}{dx} = 6y^2x$$

with

$$y(1) = \frac{1}{25}$$

5. Solve the following IVP and find the interval of validity for the solution

$$y' = \frac{3x^2 + 4x - 4}{2y - 4}$$

with

$$y(1) = 3$$

6. Show that $y = c_1e^{2x} + c_2xe^{2x}$ is the general solution of

$$y'' - 4y' + 4y = 0$$

(Hint: Show that e^{2x} and xe^{2x} are linearly independent by computing the *Wronskian*.)

7. Solve the following homogeneous equation

$$2y'' - 5y' + 3y = 0$$

8. Solve the following non-homogeneous equation

$$y'' - 5y' + 6y = xe^x$$

9. Solve

$$y'' - 2y' + 2y = 0$$

10. Solve

$$y'' + y = \frac{1}{\cos x}$$