Math Tune up Summer 2008

Exercises
Monday August 11

(1) For the following matrices, find their characteristic polynomial, eigenvalues and associated eigenvectors. Also, what is the result you obtain when you plug in the matrix in its characteristic polynomial?

(a)
$$A = \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$$

(b)
$$B = \begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix}$$

(c)
$$C = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

(2) The integral $T : p \mapsto \int p(x)\ dx$ with constant of integration equal to 0 is a linear transformation from $\mathcal{P}_3$ into $\mathcal{P}_4$.

Find the matrix associated with T relative to the canonical bases of $\mathcal{P}_3$ and $\mathcal{P}_4$, i.e., $B = \{1, x, x^2, x^3\}$ and $F = \{1, x, x^2, x^3, x^4\}$, respectively.

(3) Lagrange Polynomial Interpolation.
In engineering sometimes we want to find the exact form of a function. A polynomial is an excellent candidate. Given a set of points $(-2, 7), (-1, 4), (0, 1), (1, -2)$, find a polynomial of degree 4 that admits these points. In addition, try to give a general process of Lagrange Polynomial Interpolation with n points. Is the degree of the polynomial definitely n?