Putzer's Method via the \mathcal{Z} -Transform

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Outline

Putzer's Method via the \mathcal{Z} -Transform

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Difference Equations

Difference equations (or recurrence relations) are the discrete equivalent of a differential equation.

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A famous example is the equation that defines the Fibonacci numbers.

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Difference Equations

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A famous example is the equation that defines the Fibonacci numbers.

$$f(n) = f(n-2) + f(n-1), \quad f(0) = 0, \quad f(1) = 1$$

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The general form of first order linear difference equations is

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$$y(k+1) = ay(k) + f(k)$$

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$$y(k+1) = ay(k) + f(k)$$

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 $y(0) = y_0$

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$$y(k+1) = ay(k) + f(k)$$

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The solution is

The general form of first order linear difference equations is

$$y(k+1) = ay(k) + f(k)$$

f(k) = 0

$$y(0)=y_0$$

The solution is

$$y(k)=a^ky_0$$

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$$y(1) = ay(0)$$
(1)

$$y(2) = ay(1)$$
(2)

$$y(k) = ay(k - 1)$$
(3)

$$\prod_{i=1}^{k} y(i) = \prod_{i=0}^{k-1} ay(i)$$
(4)
(5)

$$y(k) = a^{k}y(0) = a^{k}y_{0}$$
(6)

Putzer's Method via the Z-Transform

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$$\begin{array}{l} y(2) = ay(1) \\ \vdots \end{array} \tag{2}$$

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Putzer's Method via the Z-Transform

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Putzer's Method via the Z-Transform

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The population of the American bison is categorized into newborns, yearlings, and adults after k years.

Every year the newborns are 42% of the adults from the previous year.

$$y_1(k+1) = 0.42y_3(k)$$

• Every year 60% of the newborns become yearlings.

 $y_2(k+1) = 0.60y_1(k)$

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Every year 75% of the yearlings become adults.

• Every year 95% of the adults survive.

 $y_3(k+1) = 0.75y_2 + 0.95y_3(k)$

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Putzer's Method via the *Z*-Transform

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$$y_3(k+1) = 0.75y_2 + 0.95y_3(k)$$

$$y_1(k+1) = 0.42y_3(k), \quad y_1(0) = \alpha$$
$$y_2(k+1) = 0.6y_1(k), \quad y_2(0) = \beta$$
$$y_3(k+1) = 0.75y_2(k) + 0.9y_3(k), \quad y_3(0) = \gamma$$

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$$\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ y_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{bmatrix}$$
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Putzer's Method via the *Z*-Transform

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$$\begin{bmatrix} y_{1}(0) \\ y_{2}(0) \\ y_{3}(0) \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$\mathbf{y}(k+1) = A\mathbf{y}(k), \quad \mathbf{y}(0) = \mathbf{y}_{\mathbf{0}}$$

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$$y(k+1) = Ay(k), \quad y(0) = y_0$$

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The solution to the system of difference equations is

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$$\mathbf{y}(k+1) = A\mathbf{y}(k), \quad \mathbf{y}(0) = \mathbf{y}_{\mathbf{0}}$$

The solution to the system of difference equations is

$$\mathbf{y}(k) = A^k \mathbf{y}(0)$$

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$$A = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix}$$
$$A^{2} = \begin{bmatrix} 0 & 0.315 & 0.399 \\ 0 & 0 & 0.252 \\ 0.45 & 0.7125 & 0.9025 \end{bmatrix}$$

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- Finding powers of *A* is difficult.
- We need other tools.

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$$A^{2} = \begin{bmatrix} 0 & 0.315 & 0.399 \\ 0 & 0 & 0.252 \\ 0.45 & 0.7125 & 0.9025 \end{bmatrix}$$

The k^{th} power of A is the percentages after k years.

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Putzer's Method via the Z-Transform

$\mathcal{Z} ext{-}Transform$

The definition of the \mathcal{Z} -Transform is

$$\mathcal{Z}\{y(k)\}(z) = \sum_{k=0}^{\infty} \frac{y(k)}{z^k} = Y(z)$$

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\mathcal{Z} -Transform	
\mathcal{Z} { 1 }(<i>z</i>)	$\frac{z}{z-1}$
$\mathcal{Z}{a^k}(z)$	$\frac{z}{z-a}$
$\mathcal{Z}\{y(k+1)\}(z)$	$z\mathcal{Z}\{y(k)\}-y(0)z$

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\mathcal{Z} -Transform Example

\mathcal{Z} -Transform	
\mathcal{Z} {1}(z)	$\frac{z}{z-1}$
$\mathcal{Z}{a^k}(z)$	$\frac{z}{z-a}$
$\mathcal{Z}\{y(k+1)\}(z)$	$z\mathcal{Z}\{y(k)\}-y(0)z$

 $y(k+1) - 5y(k) + 6 = 0, \quad y(0) = 0$

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Putzer's Method via the Z-Transform

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Putzer's Method via the \mathcal{Z} -Transform

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$\mathcal{Z}\{y(k+1)\}(z)$	$z\mathcal{Z}\{y(k)\}-y(0)z$

 $\mathcal{Z}\{y(k+1)\}(z) - 5\mathcal{Z}\{y(k)\}(z) + 6\mathcal{Z}\{1\}(z) = 0$

$$zY(z) - y(0)z - 5Y(z) + 6\frac{z}{z-1} = 0$$

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Putzer's Method via the *Z*-Transform

$\mathcal{Z}\text{-}\textsc{Transform}$ Example

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\mathcal{Z} { 1 }(<i>z</i>)	$\frac{z}{z-1}$
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$\mathcal{Z}\{y(k+1)\}(z)$	$z\mathcal{Z}\{y(k)\}-y(0)z$

$$zY(z) - y(0)z - 5Y(z) + 6\frac{z}{z-1} = 0$$

$$(z-5)Y(z) = -6\frac{z}{z-1}$$

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Putzer's Method via the Z-Transform

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$$(z-5)Y(z) = -6\frac{z}{z-1}$$

$$Y(z) = -6\frac{z}{(z-1)(z-5)}$$

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Putzer's Method via the *Z*-Transform

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$\mathcal{Z}\{y(k+1)\}(z)$	$z\mathcal{Z}\{y(k)\}-y(0)z$

$$Y(z) = -6\frac{z}{(z-1)(z-5)}$$
$$Y(z) = -6z\left(\frac{-1}{4(z-1)} + \frac{1}{4(z-5)}\right)$$

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Putzer's Method via the Z-Transform

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$\mathcal{Z}{y(k+1)}(z)$	$z\mathcal{Z}\{y(k)\}-y(0)z$

$$Y(z) = -6z \left(\frac{-1}{4(z-1)} + \frac{1}{4(z-5)}\right)$$
$$Y(z) = \frac{3z}{2(z-1)} - \frac{3z}{2(z-5)}$$

Putzer's Method via the \mathcal{Z} -Transform

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Putzer's Method via the Z-Transform

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$$Y(z) = rac{3z}{2(z-1)} - rac{3z}{2(z-5)}$$
 $\mathcal{Z}^{-1}\{Y(z)\} = rac{3}{2} - rac{3}{2}5^k$

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$\mathcal{Z}\text{-}\mathsf{Transform}$ on a System of Difference Equations

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Apply the \mathcal{Z} -transform to every value in the matrix.

Putzer's Method via the \mathcal{Z} -Transform

$\mathcal{Z}\text{-}\mathsf{Transform}$ on a System of Difference Equations

Apply the \mathcal{Z} -transform to every value in the matrix.

$$\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ y_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{bmatrix}$$

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\mathcal{Z} -Transform on a System of Difference Equations

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$$zY_1(z) - zy_1(0) = Y_3(z)$$
$$zY_2(z) - zy_2(0) = 0.60Y_1(z)$$
$$zY_3(z) - zy_3(0) = 0.75Y_2(z) + 0.95Y_3(z)$$

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\mathcal{Z} -Transform on a System of Difference Equations

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Solve using known methods.

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Putzer's Method

Putzer's algorithm solves $\mathbf{y}(k+1) = A\mathbf{y}(k)$.

- ► A, n xn coefficient matrix
- $\mathbf{y}(k+1)$ and $\mathbf{y}(k)$,*n*-component column vectors

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- A, n xn coefficient matrix
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$$\mathbf{y}(k) = \sum_{i=0}^{n-1} c_{i+1}(k) M_i \mathbf{y}(0) = A^k \mathbf{y}(0)$$

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Putzer's algorithm solves $\mathbf{y}(k+1) = A\mathbf{y}(k)$.

- A, n xn coefficient matrix
- $\mathbf{y}(k+1)$ and $\mathbf{y}(k)$,*n*-component column vectors

$$\mathbf{y}(k) = \sum_{i=0}^{n-1} c_{i+1}(k) M_i \mathbf{y}(0) = A^k \mathbf{y}(0)$$
$$M_0 = I$$
$$M_i = (\mathbf{A} - \lambda_i I) (\mathbf{A} - \lambda_{i-1} I) \dots (\mathbf{A} - \lambda_1 I)$$

Putzer's Algorithm

$$c_{1}(k+1) = \lambda_{1}c_{1}(k), \quad c_{1}(0) = 1$$

$$c_{2}(k+1) = \lambda_{2}c_{2}(k) + c_{1}(k), \quad c_{2}(0) = 0$$

$$c_{3}(k+1) = \lambda_{3}c_{3}(k) + c_{2}(k), \quad c_{3}(0) = 0$$

$$\vdots$$

$$c_{n}(k+1) = \lambda_{n}(k) + c_{n-1}(k), \quad c_{n}(0) = 0$$

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$$\mathbf{y}(k+1) = A\mathbf{y}(k), \quad A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$p_A(\lambda) = |A - \lambda I| = \begin{vmatrix} -\lambda & 1 \\ -2 & -3 - \lambda \end{vmatrix} = (\lambda + 1)(\lambda + 2) = 0$$

$$\lambda_1 = -1, \quad \lambda_2 = -2$$

$$M_0 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad M_1 = \begin{bmatrix} A - \lambda_1 I \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$$

Putzer's Method via the \mathcal{Z} -Transform

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$$\mathbf{y}(k+1) = A\mathbf{y}(k), \quad A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

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Putzer's Method via the \mathcal{Z} -Transform

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Putzer's Method via the \mathcal{Z} -Transform

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Putzer's Method via the Z-Transform

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$$c_{1}(k+1) = \lambda_{1}c_{1}(k), \quad c_{1}(0) = 1$$
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$$c_{1}(k+1) = -c_{1}(k)$$
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$$\mathcal{Z}\{c_{1}(k+1)\}(z) = -\mathcal{Z}\{c_{1}(k)\}(z)$$
(9)

$$C_{1}(z) - zc_{1}(0) = -C_{1}(z)$$
(10)

$$C_{1}(z) = \frac{z}{z+1}$$
(11)

$$\mathcal{Z}^{-1}\{C_{1}(z)\} = (-1)^{k}$$
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Putzer's Method via the \mathcal{Z} -Transform

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$$zC_2(z) - zc_2(0) = -2C_2(z) + \frac{z}{z+1}$$
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$$C_2(z) = \frac{z}{(z+1)(z+2)}$$
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$$\mathcal{Z}^{-1}\{C_2(z)\} = (-1)^k - (-2)^k \tag{18}$$

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Putzer's Method via the \mathcal{Z} -Transform

$\mathbf{y}(k) = (c_1(k)M_0 + c_2(k)M_1)\mathbf{y}(0) =$

$$\left((-1)^{k} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + ((-1)^{k} - (-2)^{k}) \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}\right) \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

Final solution:

$$\begin{bmatrix} 2(-1)^{k} - (-2)^{k} & (-1)^{k} - (-2)^{k} \\ -2(-1)^{k} + 2(-2)^{k} & -(-1)^{k} + 2(-2)^{k} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

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Putzer's Method via the \mathcal{Z} -Transform

$$\mathbf{y}(k) = (c_1(k)M_0 + c_2(k)M_1)\mathbf{y}(0) = \\ \left((-1)^k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + ((-1)^k - (-2)^k) \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \right) \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

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Putzer's Method via the Z-Transform

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Putzer's Method via the Z-Transform

$$A^{k} = \begin{bmatrix} 2(-1)^{k} - (-2)^{k} & (-1)^{k} - (-2)^{k} \\ -2(-1)^{k} + 2(-2)^{k} & -(-1)^{k} + 2(-2)^{k} \end{bmatrix}$$

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Putzer's Method via the \mathcal{Z} -Transform

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$$A^{0} = \begin{bmatrix} 2(-1)^{0} - (-2)^{0} & (-1)^{0} - (-2)^{0} \\ -2(-1)^{0} + 2(-2)^{0} & -(-1)^{0} + 2(-2)^{0} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = A^{0}$$

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Putzer's Method via the \mathcal{Z} -Transform

$$A^{k} = \begin{bmatrix} 2(-1)^{k} - (-2)^{k} & (-1)^{k} - (-2)^{k} \\ -2(-1)^{k} + 2(-2)^{k} & -(-1)^{k} + 2(-2)^{k} \end{bmatrix}$$
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$$A^{1} = \begin{bmatrix} 2(-1)^{1} - (-2)^{1} & (-1)^{1} - (-2)^{1} \\ -2(-1)^{1} + 2(-2)^{1} & -(-1)^{1} + 2(-2)^{1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = A^{1}$$

Putzer's Method via the \mathcal{Z} -Transform

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$$A^{k} = \begin{bmatrix} 2(-1)^{k} - (-2)^{k} & (-1)^{k} - (-2)^{k} \\ -2(-1)^{k} + 2(-2)^{k} & -(-1)^{k} + 2(-2)^{k} \end{bmatrix}$$
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$$A^{2} = \begin{bmatrix} 2(-1)^{2} - (-2)^{2} & (-1)^{2} - (-2)^{2} \\ -2(-1)^{2} + 2(-2)^{2} & -(-1)^{2} + 2(-2)^{2} \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 6 & -7 \end{bmatrix} = A^{2}$$

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The rest of the solutions can be checked inductively.

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Need new closed form solution.

- Putzer's recursion is tedious.
- \triangleright *Z*-transform can eliminate recursion.

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$$A^{k} = \sum_{i=0}^{n-1} c_{i+1}(k) M_{i}$$

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- Putzer's recursion is tedious.
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$$A^{k} = \sum_{i=0}^{n-1} c_{i+1}(k) M_{i}$$
$$M_{0} = I$$
$$M_{i} = (A - \lambda_{i}I)(A - \lambda_{i-1}I) \dots (A - \lambda_{1}I)$$

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- Putzer's recursion is tedious.
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$$M_{0} = I$$
$$M_{i} = (A - \lambda_{i}I)(A - \lambda_{i-1}I) \dots (A - \lambda_{1}I)$$
$$c_{i}(k) = \mathcal{Z}^{-1} \left\{ C_{i}(z) \right\} = \mathcal{Z}^{-1} \left\{ \frac{z}{(z - \lambda_{i})(z - \lambda_{i-1}) \dots (z - \lambda_{1})} \right\}$$

$$c_{1}(k+1) = \lambda_{1}c_{1}(k), \quad c_{1}(0) = 1$$
(19)

$$\mathcal{Z}\{c_{1}(k+1)\}(z) = \lambda_{1}\mathcal{Z}\{c_{1}(k)\}(z)$$
(20)

$$zC_{1}(z) - zc_{1}(0) = \lambda_{1}C_{1}(z)$$
(21)

$$zC_{1}(z) - z = \lambda_{1}C_{1}(z)$$
(22)

$$C_{1}(z) = -\frac{z}{z}$$
(23)

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$$c_1(k+1) = \lambda_1 c_1(k), \quad c_1(0) = 1$$
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$$c_{2}(k+1) = \lambda_{2}c_{2}(k) + c_{1}(k), \quad c_{2}(0) = 0$$
(24)
$$c_{2}(k+1)\}(z) = \lambda_{2}\mathcal{Z}\{c_{2}(k)\}(z) + \mathcal{Z}\{c_{1}(k)\}(z)$$
(25)

$$zC_2(z) - zc_2(0) = \lambda_2 C_2(z) + \frac{z}{z - \lambda_1}$$
 (26)

$$zC_2(z) = \lambda_2 C_2(z) + \frac{z}{z - \lambda_1}$$
(27)

$$C_2(z) = \frac{z}{(z - \lambda_2)(z - \lambda_1)}$$
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$$G_2(z) = \frac{z}{(z - \lambda_2)(z - \lambda_1)}$$
(28)

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$$c_2(k+1) = \lambda_2 c_2(k) + c_1(k), \quad c_2(0) = 0$$
 (24)

$$\mathcal{Z}\{c_2(k+1)\}(z) = \lambda_2 \mathcal{Z}\{c_2(k)\}(z) + \mathcal{Z}\{c_1(k)\}(z)$$
(25)

$$zC_2(z) - zc_2(0) = \lambda_2 C_2(z) + \frac{z}{z - \lambda_1}$$
 (26)

$$zC_2(z) = \lambda_2 C_2(z) + \frac{z}{z - \lambda_1}$$
 (27)

$$C_2(z) = \frac{z}{(z - \lambda_2)(z - \lambda_1)}$$
(28)

$$c_2(k+1) = \lambda_2 c_2(k) + c_1(k), \quad c_2(0) = 0$$
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$$\mathcal{Z}\{c_2(k+1)\}(z) = \lambda_2 \mathcal{Z}\{c_2(k)\}(z) + \mathcal{Z}\{c_1(k)\}(z)$$
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$$zC_2(z) - zc_2(0) = \lambda_2 C_2(z) + \frac{z}{z - \lambda_1}$$
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$$C_2(z) = \frac{z}{(z - \lambda_2)(z - \lambda_1)}$$
(28)

$$c_3(k+1) = \lambda_3 c_3(k) + c_2(k), \quad c_3(0) = 0$$
 (29)

$$\mathcal{Z}\{c_3(k+1)\}(z) = \lambda_3 \mathcal{Z}\{c_3(k)\}(z) + \mathcal{Z}\{c_3(k)\}(z)$$
(30)

$$zC_3(z) - zc_3(0) = \lambda_3 C_3(z) + \frac{z}{(z - \lambda_2)(z - \lambda_1)}$$
(31)

$$zC_3(z) = \lambda_3 C_3(z) + \frac{z}{(z - \lambda_2)(z - \lambda_1)}$$
 (32)

$$C_3(z) = \frac{z}{(z - \lambda_3)(z - \lambda_2)(z - \lambda_1)}$$
(33)

$$c_3(k+1) = \lambda_3 c_3(k) + c_2(k), \quad c_3(0) = 0$$
 (29)

$$\mathcal{Z}\{c_3(k+1)\}(z) = \lambda_3 \mathcal{Z}\{c_3(k)\}(z) + \mathcal{Z}\{c_3(k)\}(z)$$
(30)

$$zC_{3}(z) - zc_{3}(0) = \lambda_{3}C_{3}(z) + \frac{z}{(z - \lambda_{2})(z - \lambda_{1})}$$
(31)

$$z\mathcal{C}_3(z) = \lambda_3\mathcal{C}_3(z) + \frac{1}{(z-\lambda_2)(z-\lambda_1)}$$
(32)

$$C_3(z) = \frac{z}{(z - \lambda_3)(z - \lambda_2)(z - \lambda_1)}$$
(33)

$$c_3(k+1) = \lambda_3 c_3(k) + c_2(k), \quad c_3(0) = 0$$
 (29)

$$\mathcal{Z}\{c_3(k+1)\}(z) = \lambda_3 \mathcal{Z}\{c_3(k)\}(z) + \mathcal{Z}\{c_3(k)\}(z)$$
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 (32)

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(33)

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Defining the New Coefficients

$$c_{1}(k) = \mathcal{Z}^{-1}\{C_{1}(z)\} = \mathcal{Z}^{-1}\left\{\frac{z}{z-\lambda_{1}}\right\}$$

$$c_{2}(k) = \mathcal{Z}^{-1}\{C_{2}(z)\} = \mathcal{Z}^{-1}\left\{\frac{z}{(z-\lambda_{2})(z-\lambda_{1})}\right\}$$

$$c_{3}(k) = \mathcal{Z}^{-1}\{C_{3}(z)\} = \mathcal{Z}^{-1}\left\{\frac{z}{(z-\lambda_{3})(z-\lambda_{2})(z-\lambda_{1})}\right\}$$

$$\vdots$$

$$c_{n}(k) = \mathcal{Z}^{-1}\{C_{n}(z)\} = \mathcal{Z}^{-1}\left\{\frac{z}{(z-\lambda_{n})(z-\lambda_{n-1})\dots(z-\lambda_{1})}\right\}$$

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$$\mathbf{y}(k+1) = A\mathbf{y}(k), \quad A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$p_A(\lambda) = |A - \lambda I| = \begin{vmatrix} -\lambda & 1 \\ -2 & -3 - \lambda \end{vmatrix} = (\lambda + 1)(\lambda + 2) = 0$$

$$\lambda_1 = -1, \quad \lambda_2 = -2$$

$$M_0 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad M_1 = \begin{bmatrix} A - \lambda_1 I \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$$

Putzer's Method via the \mathcal{Z} -Transform

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$$\mathbf{y}(k+1) = A\mathbf{y}(k), \quad A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

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Putzer's Method via the \mathcal{Z} -Transform

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Putzer's Method via the \mathcal{Z} -Transform

$$\mathbf{y}(k+1) = A\mathbf{y}(k), \quad A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

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$$c_{1}(k) = \mathcal{Z}^{-1}\left\{\frac{z}{z+1}\right\} = (-1)^{k}$$

$$c_{2}(k) = \mathcal{Z}^{-1}\left\{\frac{z}{(z+1)(z+2)}\right\} = (-1)^{k} - (-2)^{k}$$

$$A^{k} = c_{1}(k)M_{0} + c_{2}(k)M_{1} = \left((-1)^{k} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + ((-1)^{k} - (-2)^{k}) \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}\right)$$

Final solution:

$$egin{bmatrix} 2(-1)^k - (-2)^k & (-1)^k - (-2)^k \ -2(-1)^k + 2(-2)^k & -(-1)^k + 2(-2)^k \end{bmatrix}$$

Putzer's Method via the \mathcal{Z} -Transform

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$$A^{k} = c_{1}(k)M_{0} + c_{2}(k)M_{1} = \left((-1)^{k} \begin{bmatrix} 1 & 0\\ 0 & 1 \end{bmatrix} + ((-1)^{k} - (-2)^{k}) \begin{bmatrix} 1 & 1\\ -2 & -2 \end{bmatrix}$$

Final solution:

$$\begin{bmatrix} 2(-1)^k - (-2)^k & (-1)^k - (-2)^k \\ -2(-1)^k + 2(-2)^k & -(-1)^k + 2(-2)^k \end{bmatrix}$$

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Final solution:

$$\begin{bmatrix} 2(-1)^k - (-2)^k & (-1)^k - (-2)^k \\ -2(-1)^k + 2(-2)^k & -(-1)^k + 2(-2)^k \end{bmatrix}$$

Thank You!

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Putzer's Method via the \mathcal{Z} -Transform

$$y_1(k+1) = 0.42y_3(k)$$
$$y_2(k+1) = 0.6y_1(k)$$
$$y_3(k+1) = 0.75y_2(k) + 0.9y_3(k)$$

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Putzer's Method via the Z-Transform

$$y_1(k+1) = 0.42y_3(k)$$
$$y_2(k+1) = 0.6y_1(k)$$
$$y_3(k+1) = 0.75y_2(k) + 0.9y_3(k)$$

$$ZY_1(z) - Zy_1(0) = Y_3(z)$$
$$ZY_2(z) - Zy_2(0) = 0.60Y_1(z)$$
$$ZY_3(z) - Zy_3(0) = 0.75Y_2(z) + 0.95Y_3(z)$$

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Putzer's Method via the Z-Transform

$$\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ y_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{bmatrix}$$

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Putzer's Method via the \mathcal{Z} -Transform

$$\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ y_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{bmatrix}$$
$$z \begin{bmatrix} Y_1(z) \\ Y_2(z) \\ Y_3(z) \end{bmatrix} - z \begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} Y_1(z) \\ Y_2(z) \\ Y_3(z) \end{bmatrix}$$

Putzer's Method via the \mathcal{Z} -Transform

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$$\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ y_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{bmatrix}$$
$$z \begin{bmatrix} Y_1(z) \\ Y_2(z) \\ Y_3(z) \end{bmatrix} - z \begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} Y_1(z) \\ Y_2(z) \\ Y_3(z) \end{bmatrix}$$

 $z\mathbf{Y}(z) - z\mathbf{Y}(z) = A\mathbf{Y}(z)$

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Putzer's Method via the \mathcal{Z} -Transform

$$\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ y_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{bmatrix}$$
$$z \begin{bmatrix} Y_1(z) \\ Y_2(z) \\ Y_3(z) \end{bmatrix} - z \begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} Y_1(z) \\ Y_2(z) \\ Y_3(z) \end{bmatrix}$$

 $z\mathbf{Y}(z) - z\mathbf{Y}(z) = A\mathbf{Y}(z)$

 $(zI - A)\mathbf{Y}(z) = z\mathbf{Y}(z)$

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Putzer's Method via the Z-Transform

$$\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ y_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{bmatrix}$$
$$z \begin{bmatrix} Y_1(z) \\ Y_2(z) \\ Y_3(z) \end{bmatrix} - z \begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.42 \\ 0.60 & 0 & 0 \\ 0 & 0.75 & 0.95 \end{bmatrix} \begin{bmatrix} Y_1(z) \\ Y_2(z) \\ Y_3(z) \end{bmatrix}$$

 $z\mathbf{Y}(z)-z\mathbf{Y}(z)=A\mathbf{Y}(z)$

 $(zI - A)\mathbf{Y}(z) = z\mathbf{Y}(z)$

$$\mathbf{Y}(z) = z(zI - A)^{-1}\mathbf{Y}(0)$$

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$$z(zI - A)^{-1} = \sum_{j=0}^{n-1} R_{j+1}(z)P_j$$

Putzer's Method via the \mathcal{Z} -Transform

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$$z(zI - A)^{-1} = \sum_{j=0}^{n-1} R_{j+1}(z)P_j$$

$$\frac{1}{z}(zI - A)R_{j+1}(s)P_j = \frac{1}{z}(((zI - \lambda_{j+1}I) - (A - \lambda_j + 1I))R_{j+1}(s)R_{j+1}I) = \frac{1}{z}((z - \lambda_{j+1})R_{j+1}(s)P_j - R_{j+1}(s)(A - \lambda_j + 1I)) = \frac{1}{z}(R_{j+1}(s)P_j - R_{j+1}(s)P_{j+1})$$

$$z(zI - A)Y(s) = \sum_{j=0}^{n-1} \frac{(zI - A)R_{j+1}(z)P_j}{z}$$

Putzer's Method via the \mathcal{Z} -Transform

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