ORDERED STRUCTURES IN GEOMETRY AND ANALYSIS

Titles and Abstracts

Gerhard Gierz:

Title:

Abstract:

Joachim Hilgert:

Title: Spectral properties of convex bodies

Abstract: Motivated by the description of state spaces in quantum mechanics we discuss spectral properties of finite dimensional convex bodies. This means properties referring to the ways in which a state can be obtained as a convex combination of pure states. If the pure states can be chosen distinguishable by measurements one calls the state space spectral. The main result is that spectral state spaces which is in addition strongly symmetric (which refers to certain transitivity properties of the symmetry group) necessarily must be a simplex or the normalized state space of a simple Euclidean Jordan algebra. This is joint work in progress with Howard Barnum.

Karl Heinrich Hofmann:

Title: The pro-Lie Theory of topological algebras modelled on $\mathbb{R}^X$ and $\mathbb{C}^X$.

Abstract: A topological vector space over $\mathbb{K} = \mathbb{R}$ or $= \mathbb{C}$ is called *weakly complete* if (and only if) it is isomorphic to $\mathbb{K}^X$ for some set X algebraically and topologically. The significance of this type of topological vector spaces is illustrated by the fact that the underlying vector space of the Lie algebra of any pro-Lie group is weakly complete. Here weakly complete real or complex associative algebras are studied because they are necessarily projective limits of finite dimensional algebras. The group of units A^{-1} of a weakly complete algebra A is a pro-Lie group with the associated topological Lie algebra A_{Lie} of A as Lie algebra and the globally defined exponential function $\exp\colon A_{\mathrm{Lie}} \to A^{-1}$ as the exponential function of A^{-1}. With each topological group, a weakly complete group algebra $\mathbb{K}[G]$ is associated functorially so that the functor $G \mapsto \mathbb{K}[G]$ is left adjoint to $A \mapsto A^{-1}$. The group algebra $\mathbb{K}[G]$ is a weakly complete Hopf algebra. If G is compact, the $\mathbb{R}[G]$ contains G as the set of grouplike elements. The category $\mathcal{H}$ of al weakly complete real Hopf algebras A with a compact group of grouplike elements whose linear span is dense in A is shown to be equivalent to the category of compact groups. The group algebra $A = \mathbb{R}[G]$ of a compact group G contains a copy of the Lie algebra $\mathfrak{L}(G)$ in A_{Lie}; it also contains a copy of the Radon measure algebra $M(G, \mathbb{R})$. The dual of the group algebra $\mathbb{R}[G]$ is the Hopf algebra $\mathcal{R}(G, \mathbb{R})$ of representative functions of G. The rather straightforward duality between vector spaces and weakly complete vector spaces thus becomes the basis of a duality $\mathcal{R}(G, \mathbb{R}) \leftrightarrow \mathbb{R}[G]$ and thus yields a new aspect of Tannaka duality.

Velimir Jurdjevic:

Extremal curves on Stiefel and Grassmann manifolds: Quasi-geodesics

Abstract: This talk will introduce a large class of left-invariant sub-Riemannian systems on Lie groups that admit explicit solutions. Their study was inspired by the search for the geometric origins of a class of curves on Stiefel manifolds, called quasi-geodesics, that are important in interpolation theory. I will show that quasi-geodesics are the projections of sub-Riemannian geodesics generated by certain left-invariant distributions on Lie groups that act transitively on each Stiefel manifold $St_k^n(V)$. These results are based on a joint work with Irina Markina and Fatima Silva-Leite.

References

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[JKL] V. Jurdjevic, K. Krakowski K, and F. Silva Leite: *The geometry of quasi-geodesics on Stiefel manifolds*, The proceedings of the 13th APCA International Conference on Control and Soft Computing, Ponta Delgada, Sao Miguel Island, Azores, Portugal, June 2018

[KM] K. Krakowski, L. Machado, F. Silva Leite, and J. Batista: *A modified Casteljau algorithm to solve interpolation problems on Stiefel manifolds*, Journal of Computational and Applied Mathematics, **311** (2017), 84–99.

Sejong Kim:

Title: Parameterized Wasserstein mean of positive definite matrices

Abstract: From an optimization problem for the sandwiched quasi-relative entropy (the parameterized version of fidelity), a new parameterized matrix mean on the cone of positive definite matrices has been recently introduced. This mean generalizes the well-known Wasserstein mean (the least squares mean for the Bures-Wasserstein metric), so we call it the parameterized Wasserstein mean. We see in this talk several interesting properties of the parameterized Wasserstein mean including the determinantal inequality and bounds for the Loewner order and operator norm. Furthermore, we find relationships of the parameterized Wasserstein mean with other matrix means and show the majorization property with the Cartan mean (the least squares mean for the Riemannian trace metric).

Yongdo Lim:

Title: An extremal problem for positive semidefinite matrices

Abstract: We consider an $m \times m$ real symmetric matrix $\mathbf{M}_{\mathbf{a}}(x)$ with $a_1, \ldots, a_m$ on the main diagonal and x in all off-diagonal positions, where $\mathbf{a} = (a_1, \ldots, a_m)$ is a given m-tuple of positive real numbers and consider the extremal problem of finding the minimum and maximum of x where $\mathbf{M}_{\mathbf{a}}(x)$ is positive semidefinite. We show that the polynomial $\det\mathbf{M}_{\mathbf{a}}(x)$ in variable x has only real roots with a unique negative root and that $\mathbf{M}_{\mathbf{a}}(x)$ is positive semidefinite if and only if x lies in the closed interval determined by the negative and smallest positive roots. The negative and smallest positive root maps over m-tuples of positive real numbers contract the Thompson metric and induce new multivariate means of positive real numbers satisfying the monotonicity, homogeneity, joint concavity and super-multiplicativity. In particular, the smallest positive root map extends to such a mean of infinite variable of positive real numbers that realizes the limits for

decreasing sequences, and it eventually gives rise to a shift invariant mean of bounded sequences. We will present some related problems on algebraic curves, Hadamard semigroups of positive semidefiite matrices, and the classical Birkhoff ergodic theorem with the shift invariant mean.

Bin Lu:

Title: Domains and Integrals via Domains

Abstract: In this talk, we consider the problem of introducing a type of integral with constructive nature for

$$f : X \to \mathbb{R}$$

where X is a general topological space. Our goal is to develop a constructive approaches with effective computational algorithms to integration in more general setting, in contrast to the Lebesgue theory. We will discuss the domain theoretic approach to the Generalized Riemann integral introduced by Edalat. Following this, we will describe an alternative approach to this type of domain-theoretic integral.

Michael W. Mislove:

Title: Jimmie Lawson's Pearls: Theorems and Proof Techniques Others Use

Abstract: Jimmie Lawson's career is distinguished by a remarkable number of results across a range of areas, all of uniformly high quality. One aspect of Jimmie's work that I find particularly compelling is that so many of his results and his proof techniques are used by others in their own research. In this talk, I'll discuss two examples and how they affected my research.

Karl-Hermann Neeb

Titel: Finite dimensional endomorphism semigroups of standard subspaces

Absract: Let $M \subseteq B(H)$ be a von Neumann algebra with a cyclic separating unit vector Ω and the modular objects (Δ, J) obtained from the Tomita–Takesaki Theorem. Further, let $G \subseteq U(H)$ be a finite dimensional Lie group of unitary operators fixing Ω, containing the corresponding modular group $(\Delta^{it})_{t\in\mathbb{R}}$ and invariant under conjugation with the modular conjugation J. We study the subsemigroup

$$S_M := \{g \in G : gMg^{-1} \subseteq M\}$$

of those elements of G acting on M by endomorphisms.

Our approach is based on the inclusion of S_M into the subsemigroup $S_V = \{g \in G : gV \subseteq V\}$ for the standard subspace $V := \overline{\{m\Omega : m = m^* \in M\}}$. The semigroup S_V is analyzed in terms of antiunitary representations and their analytic extension to semigroups of the form $G\exp(iC)$, where $C \subseteq \mathfrak{g}$ is a suitable closed convex cone. In particular, we determine the Lie wedge $L(S_V) = \{x \in L(G) : \exp(\mathbb{R}_+x) \subseteq S_V\}$ of generators of its one-parameter subsemigroups.

The Lie wedge $L(S_V)$ is contained in a 3-graded Lie subalgebra in which it can be determined explicitly in terms of the involution τ of $\mathfrak{g}$ induced by J, the generator h of the modular group, and the positive cone of the corresponding representation.

Miklós Pálfia:

Title: Sturm's law of large numbers for the L^1-Karcher mean of positive operators

Abstract: Firstly we briefly review some available versions of the strong law of large numbers in Banach spaces and nonlinear extensions provided by Sturm in CAT(0) metric spaces. Sturm's 2001 L^2-result was directly applied to the case of the geometric (also called Karcher) mean of positive matrices, thus it suggests a natural formulation of the law for positive operators. However there are serious obstacles to overcome to prove the law in the infinite dimensional case. We propose to use a recently established gradient flow theory by Lim-P for the Karcher mean of positive operators and a stochastic proximal point approximation to prove the L^1-strong law of large numbers for the Karcher mean in the operator case.

Tin-Yau Tam

Title: Geometric mean inequalities and their generalizations

Abstract: In this talk we will discuss the geometry and inequalities associated with the geometric mean of two $n \times n$ positive definite matrices, which was introduced by W. Pusz and S. L. Woronowicz. The space of $n \times n$ positive definite matrices of determinant 1 is a Riemannian manifold and its geometry is hyperbolic. We show that geodesic convexity emerges when a natural pre-order call log majorization is introduced to the manifold. We also discuss several geometric mean inequalities and their extensions in the context of symmetric spaces.