Predicting Equities Markets using Monte Carlo Markov Chain Models & Probabilistic Neural Networks

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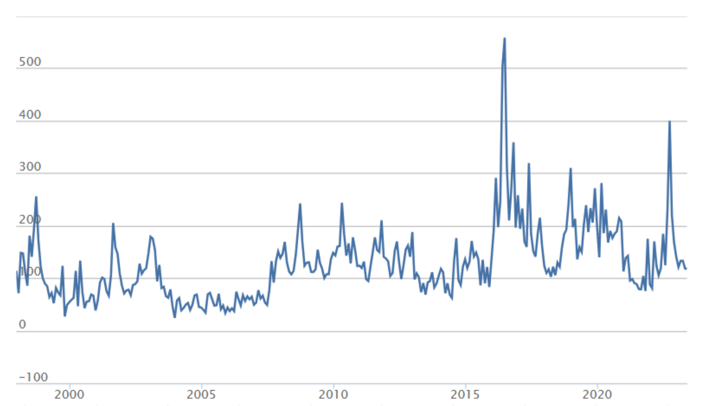
Introduction

Economic uncertainty has a lot of control over the flow of the stock market, which is why it is our goal to use S&P 500 data to run models and test empirical results to try and predict the most likely outcomes of the S&P 500 over the next two years. We do this by using a maximum likelihood estimator to estimate the transition probabilities of each state of the Markov Chain (EPU index). Then, with the use of a neural network, we use a Markov Chain Monte Carlo simulation to forecast the most likely path the EPU will take between state spaces over time. We will follow this using 2 alternative approaches of computation.

The Economic Policy Uncertainty Index

The EPU Index, created in 1997, is an index that provides an analyzable metric associated with policy-related economic uncertainty. The S&P 500 is a stock market index that tracks the prices of shares from hundreds of companies, being one of the main tools in following stock performance. Our goal is to use the United States EPU Index when analyzing how the S&P 500 reacts in times of high economic uncertainty, with theory suggesting it should fall or depreciate given that this typically correlates with stock market selloffs. The United States EPU has data running back from January 1985 through the present on a monthly basis.

The graphs below show the level of uncertainty of the EPU index and the growth of the S&P 500 over the last 25 years, respectively.



Markov Chains and The Properties of the EPU Index

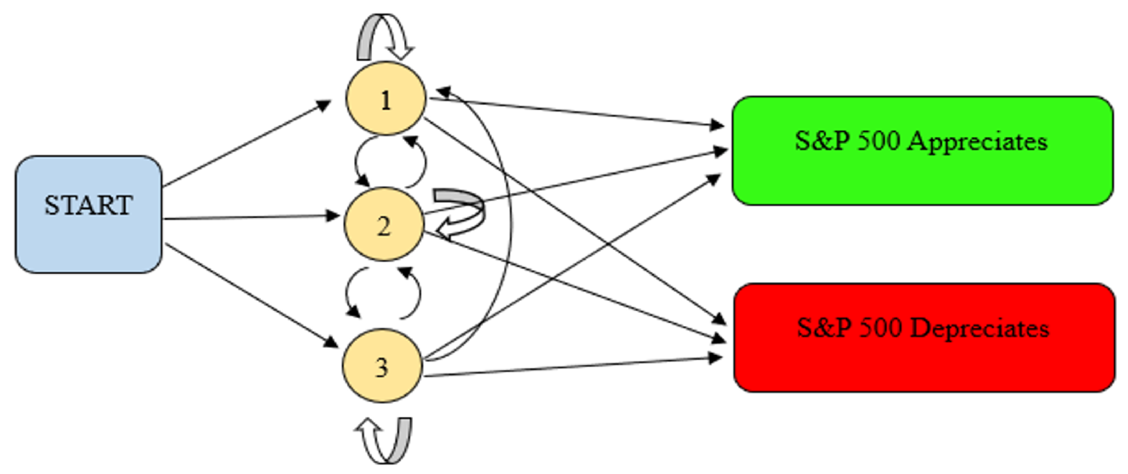
The Markov Property means that the Markov process in the future depends only on the present state and does **not** depend on past history.

The following are properties of Markov Chains that coincide with the EPU Index:

- The sums of the matrix's rows must equal **one**.
- Each probability within the matrix eventually reaches a steady state or stationary distribution as **n** becomes sufficiently large.
- The state space **(S)** of the EPU is S = {1, 2, 3}.
  - Each state space represents the top, middle, and bottom third of all observations of the EPU. State space 1 represents high economic uncertainty, state space 2 represents moderate economic uncertainty, and state space 3 represents low economic uncertainty.
- The transition probability matrix of the EPU:

$$P_{i,j} = \begin{pmatrix} 0.76 & 0.24 & 0 \\ 0.21 & 0.56 & 0.23 \\ 0.03 & 0.20 & 0.77 \end{pmatrix}$$

Creating a Neural Network using Markov Model Properties



Monte Carlo Markov Chain (MCMC) Modeling Approaches

The following are two different MCMC models we have created to approach the predictions of these equities markets. Each model is based off conditional probabilities of S&P going up or down. The first layer is the same for both models as it runs MCMC simulations of the projected EPU sequence using the last EPU state and the EPU transition probability matrix. The second layer has an initial similar purpose in that they both run weighted random choice simulations of associated sequences of S&P going up or down based on EPU state switches.

Model 1: Two-Layer MCMC Simulations with *Constant* Conditional Probabilities

In the second layer, each EPU switch and S&P switch has an associated average percent change based on past data, generating constant conditional probabilities. Within the MCMC, as the states go from i →j, we calculate this simulation using traditional probability methods, being the number of successful observations divided by the total observations. The model is below.

$$P(\text{S\&P 500 UP} \mid \text{EPU } i \to j) = \frac{\sum \text{Number of times S\&P goes UP and EPU } i \to j}{\sum \text{Number of times EPU } i \to j}, \forall\, i, j \in \text{S},$$

and

$$P(\text{S\&P 500 DOWN} \mid \text{EPU } i \to j) = \frac{\sum \text{Number of times S\&P goes DOWN and EPU } i \to j}{\sum \text{Number of times EPU } i \to j}, \forall\, i, j \in \text{S}.$$

Therefore, by definition,

$$P(\text{S\&P 500 UP} \mid \text{EPU } i \to j) \in [0,1],\ P(\text{S\&P 500 DOWN} \mid \text{EPU } i \to j) \in [0,1], \text{ and}$$
$$P(\text{S\&P 500 UP} \mid \text{EPU } i \to j) = 1 - P(\text{S\&P 500 DOWN} \mid \text{EPU } i \to j),\ \forall\ i,j \in \text{S}.$$

Model 2: Two-Layer MCMC Simulations with *Changing* Conditional Probabilities

The second layer, as simulations are run, conditional probabilities of S&P change based on the last value, hence the conditional probabilities are constantly changing.

For this model, let

$$\Sigma[\text{Number of times S\&P goes UP and EPU } i \to j] = p,$$
$$\Sigma[\text{Number of times S\&P goes DOWN and EPU } i \to j] = 1 - p = q,$$

and

$$\Sigma[Number of times i \to j] = z.$$

This would imply that

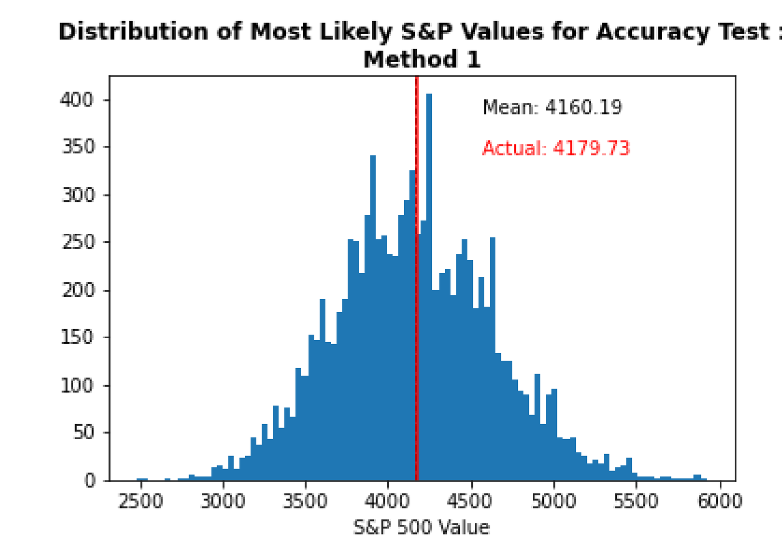
$$P(\text{S\&P 500 UP} \mid \text{EPU } i \to j) = \tfrac{p}{z} \text{ and } P(\text{S\&P 500 DOWN} \mid \text{EPU } i \to j) = \tfrac{q}{z}.$$

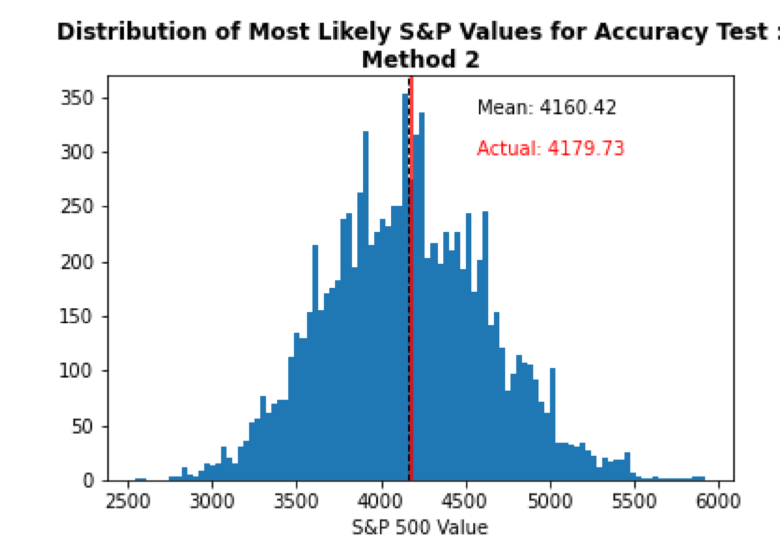
Since the probabilities in this model are changing throughout the simulation, the probabilities change at time $t + 1$ to

$$P(\text{S\&P 500 UP} \mid \text{EPU } i \to j) = \tfrac{p+1}{z+1} \text{ and } P(\text{S\&P 500 DOWN} \mid \text{EPU } i \to j) = \tfrac{q}{z+1}.$$

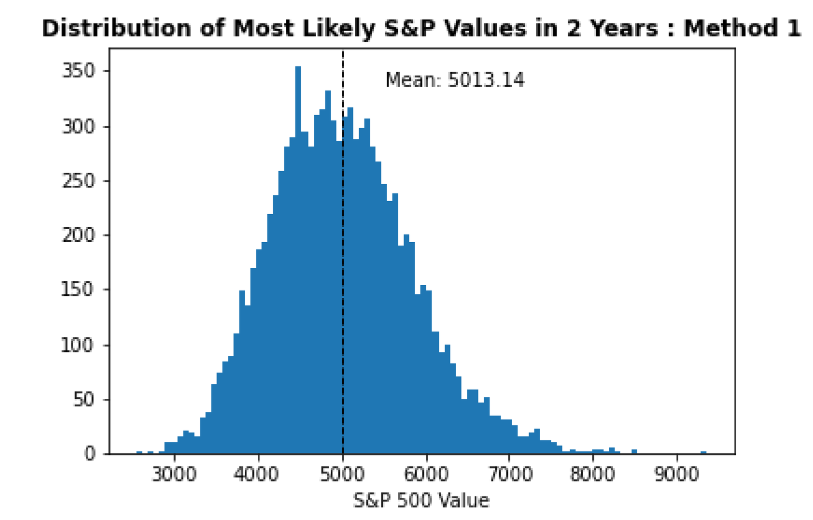
Empirical Results: Model 1 & 2 Comparisons

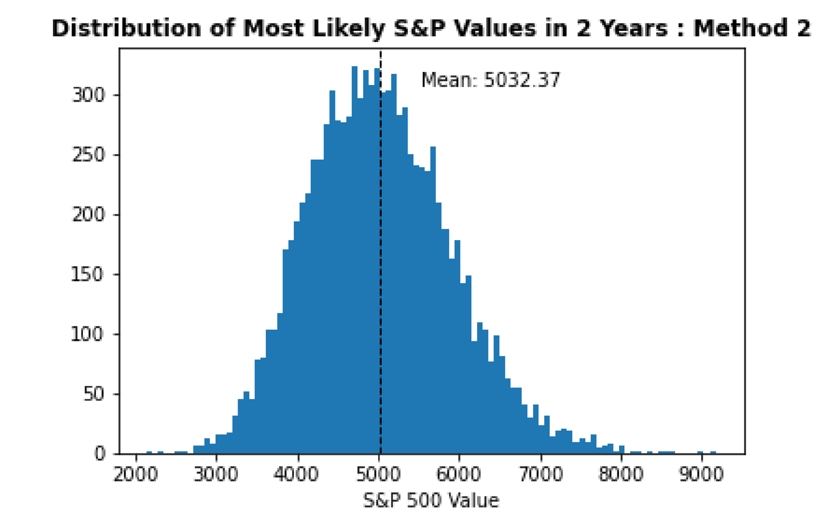
The two graphs below were generated using constructed conditional probabilities of S&P going up or down based on EPU changes of the first 450 of the 461 total observations for an accuracy test. This model ran 10,000 times and constructed the estimated value of S&P in 10 time periods. The mean of the distribution of the 10,000 simulated values was quite comparable to the actual S&P value 10 time periods later.





Next is a comparison between models 1 & 2 of their histograms of the distribution of the most likely outcomes of the S&P 500 over the next 24 time periods. These are also generated from 10,000 simulations.





Conclusion

In the last 30 years, the S&P 500 has seen an average annual growth rate of 9.5% year after year. With the use of our models, our predication for the next two years is that the S&P could see a very consistent average annual growth rate.

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