Modeling Underwater Light Transmission: Turbidity Effect

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Introduction

Underwater laser links offer high bandwidth, low latency, and phase-sensitive modulation for applications such as autonomous vehicles, sensor networks, and secure naval communications; their performance is governed by the water's optical properties: **absorption**, **scattering**, and overall **turbidity**.

In this work we analyze same-depth point-to-point links while ignoring turbulence, model attenuation with the Beer–Lambert law $T(d,\lambda) = \exp(-K_d(\lambda)\, d)$, and use Jerlov water-type diffuse attenuation coefficients $K_d(\lambda)$ to provide a physically grounded baseline for channel transmittance applicable to communication and Continuous-Variable Quantum Key Distribution performance estimates(CV$_Q KD$).

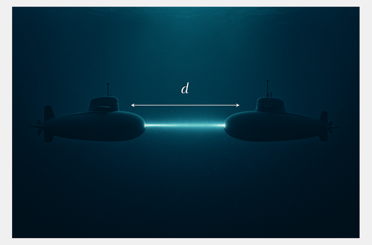

Figure 1. Schematic: Two submarines communicating at the same depth (same-depth communication).

Objectives

- **Objective A:** Model turbidity-induced variability around the Beer–Lambert baseline using a Monte-Carlo PDF approach and explore dependence on distance and heterogeneity (cells/m).
- **Objective B:** Provide plots of deterministic Beer–Lambert transmittance and stochastic Probability Distribution Function (PDF) of normalized received intensity I/I_0 across wavelengths 450–550 nm for representative Jerlov types.

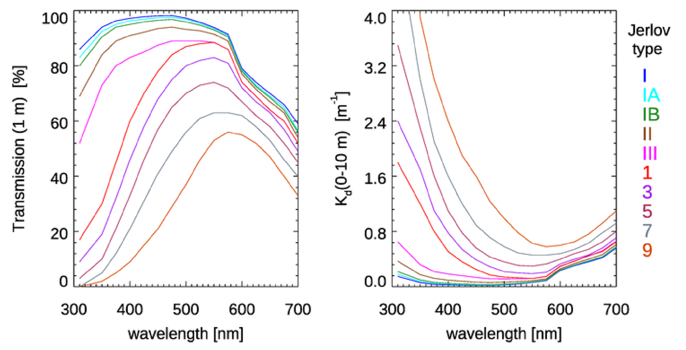

Figure 2. Jerlov K$_d$ $values. (DataSource: Jerlov, 1976)$

Background

- *Jerlov water classification*: Jerlov grouped natural waters by optical clarity (oceanic types I, IA, IB, II, III and coastal types 1–9). Each type is associated with characteristic spectral attenuation behavior; we use representative $K_d(\lambda)$ values for these types as the attenuation coefficients in the Beer–Lambert law.
- *Beer–Lambert law (same-depth)*: For a path length d and wavelength λ, $I(d,\lambda) = I_0 \exp\big(-K_d(\lambda)\, d\big)$, and the deterministic channel transmittance is

$$T_c(d,\lambda) = \frac{P_{\text{received}}}{P_{\text{transmitted}}} = \exp(-K_d(\lambda)\, d).$$

- *Absorption vs. scattering*: Particle composition controls whether attenuation is absorption-dominated (e.g., algal/phytoplankton increases μ_a) or scattering-dominated (e.g., mineral particles, CDOM increase μ_s). Both contribute to K_d but affect beam coherence differently.
- *Turbidity and heterogeneity:* Turbidity is the macroscopic manifestation of suspended particles and dissolved matter; small-scale heterogeneity can be represented statistically as fluctuations around the Beer–Lambert mean.

Method (Beer–Lambert baseline)

Assumptions.

- Same-depth geometry (transmitter and receiver at equal depth).
- Turbulence effects are ignored.
- Total attenuation described by Jerlov diffuse attenuation coefficient $K_d(\lambda)$.

Deterministic transmittance. For each Jerlov type and wavelength λ we compute $T_c(d,\lambda) = \exp\big(-K_d(\lambda)\, d\big)$,

over a range of distances d (meters). These curves form the baseline channel loss used in communication and cryptographic performance calculations.

Stochastic turbidity model . To represent small-scale turbidity variability we model the normalized received intensity

$$X = \frac{I}{I_0}$$

as a Gaussian random variable with mean $\mu = \exp(-K_d d)$ and standard deviation $\sigma = \eta(\text{cells/m}) \cdot \mu$. The PDF is

$$\text{PDF}(X) = \frac{1}{\sqrt{2\pi}\sigma} \exp\Big(-\frac{(X-\mu)^2}{2\sigma^2}\Big),$$

truncated to $X \in [0,1]$. The parameter cells/m is a tunable proxy for heterogeneity; η is a small scaling factor (e.g., 0.05–0.2). This approach reproduces PDF plots of normalized intensity while keeping the Beer–Lambert mean explicit.

Wavelength sampling. We evaluate λ in the blue–green window (450–550 nm) with 20 nm intervals to capture the spectral window of maximum penetration.

Outputs. For each Jerlov type and wavelength we produce:

- Beer–Lambert transmittance vs distance plots.
- Monte-Carlo simulation is used to observe normalized intensity I/I_0 for selected distances and cells/m values.
- Summary tables of mean transmittance vs distance.

Numerical implementation

We implement the model in Mathematica. Key steps:

1. Load representative Jerlov $K_d(\lambda)$ values (or use published tables).
2. For each λ and Jerlov type compute deterministic $T_c(d,\lambda)$.
3. Sample N realizations of $X \sim \mathcal{N}(\mu, \sigma^2)$ with $\mu = \exp(-K_d d)$ and $\sigma = \eta\mu$; clip to $[0,1]$.
4. Plot histograms (PDF) and overlay fitted Gaussian curve; repeat across distances and cells/m.

Results (expected / example panels)

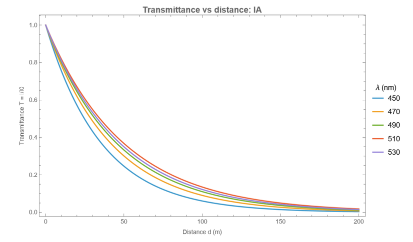

Figure 3. Distance vs Transmittance across different wavelength using Monte-Carlo simulation

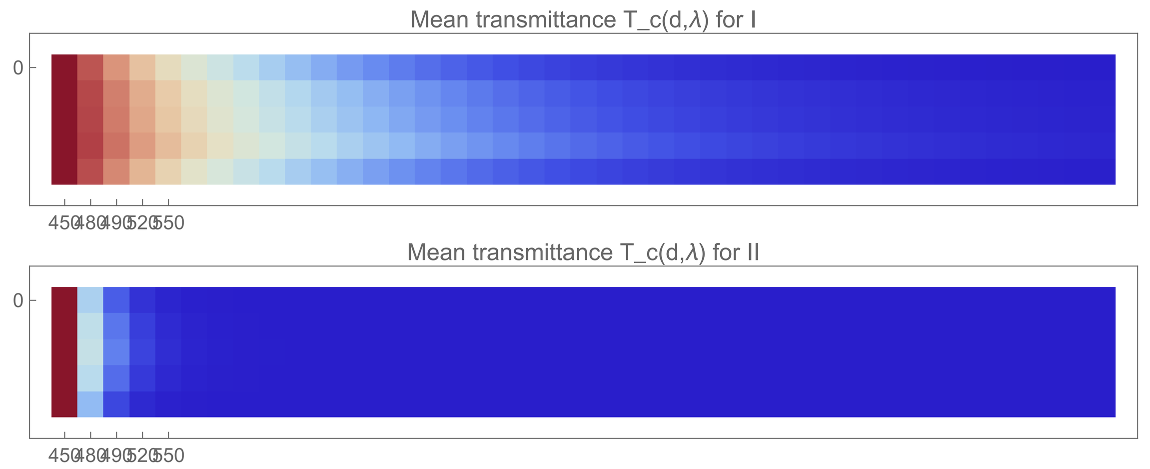

Figure 4. Heat maps for different Jerlov Water types

- **Panel 1:** Beer–Lambert transmittance vs distance for Jerlov types I, II, III at $\lambda = 480$ and 520 nm. Clear waters (Type I) show slow decay; turbid waters (Type III) decay rapidly.
- **Panel 3:** Summary heatmap of mean transmittance $T_c(d,\lambda)$ vs distance and wavelength for selected Jerlov types.

The deterministic Beer–Lambert baseline provides a direct mapping from Jerlov type to expected channel loss.

Discussion and Future work

- The Beer–Lambert + Jerlov approach yields a simple, physically grounded baseline for channel transmittance in same-depth links and is directly usable in CV-QKD key-rate calculations (channel transmittance T_c and excess noise estimates).
- Future work:Consider using a model and that differently treats absorption and scatterring coefficients; incorporate ABCD turbulence cells for beam-shape effects.

Acknowledgements

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