Section 1.1a Linear Equations

Review of Properties of Exponents

An exponent is a shorthand notation for repeated factors. For example, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ can be written as 2^5. The expression 2^5 is called an **exponential expression**. The **base** of this expression is 2, and the **exponent** is 5.

If x is a real number and n is a positive integer, then x^n is the product of n factors of x.

$$x^n = \underbrace{x \cdot x \cdot x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ factors of } x}$$

To see examples of how to use the rules of exponents listed below, watch the ***LSU Video "Exponents"*** *found on the course website.*

Product Rule for Exponents
If m and n are positive integers and a is a real number, then

$$a^m \cdot a^n = a^{m+n}.$$

Power Rule for Exponents
If m and n are positive integers and a is a real number, then

$$\left(a^m\right)^n = a^{mn}.$$

Power of a Product Rule
If n is a positive integer and a and b are real numbers, then

$$(ab)^n = a^n \cdot b^n.$$

Power of a Quotient Rule
If n is a positive integer, a and b are real numbers, and $b \neq 0$, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

Quotient Rule for Exponents
If m and n are positive integers, a is a real number, and $a \neq 0$, then

$$\frac{a^m}{a^n} = a^{m-n}.$$

Zero Exponent Rule
If b is a real number such that $b \neq 0$, then $b^0 = 1$.

Objective 1: Recognizing Linear Equations

A linear equation in one variable involves variables that are only raised to the first power.

Definition: A **linear equation in one variable** is an equation that can be written in the form $ax+b=c$, where *a*, *b*, and *c* are real numbers and $a \neq 0$.

Examples of linear equations:

$\frac{7}{2}x-4=x$ $\sqrt{5}x-3=8x-1$ $(0.7)^2x+1=6$ $2^{-1}-0.4x=0.3-5x$

Examples of non-linear equations:

$\frac{3}{x}-5=9$ $5\sqrt{x}+1=7$ $x^2+x=6$ $(0.7x)^2+x=1$ $0.3-0.2x^{-1}=2$

Review of Simplifying Algebraic Expressions

For an in depth review of this topic, watch the ***LSU Video "Simplifying Algebraic Expressions"*** *found on the course website.*

An algebraic expression containing the sum or difference of like terms can be simplified by applying the distributive property. This is called **combining like terms**.

For example, consider the expression $3x+2x$. We can use the distributive property to rewrite the sum $3x+2x$ as a product.

$$3x+2x=(3+2)x=5x$$

When simplifying an algebraic expression containing parentheses, we often use the distributive property twice, first to remove the parentheses and then to combine any like terms.

Review of Properties of Equality

For an in-depth review of this topic watch the ***LSU Video "Properties of Equality"*** *found on the course website.*

The **addition property of equality** guarantees that adding the same number to both sides of an equation creates an equation that has the same solution set as the original equation. Since subtraction is defined in terms of addition, this property also applies to subtracting the same number from both sides of an equation.

Addition Property of Equality:
If a, b, and c are real numbers and $a = b$, then $a + c = b + c$.

The **multiplication property of equality** guarantees that multiplying both sides of an equation by the same nonzero number creates an equation that has the same solution set as the original equation. Since division is defined in terms of multiplication, this property also applies to dividing both sides of an equation by the same nonzero number.

Multiplication Property of Equality:
If a, b, and c are real numbers, $c \neq 0$, and $a = b$, then $ac = bc$.

Objective 2: Solving Linear Equations with Integer Coefficients

When we solve an equation for *x*, we are looking for all values of *x* which, when substituted back into the original equation, yield a true statement. The goal here is to **isolate the variable** *x* on one side of the equation.

Review of Finding a Least Common Denominator

Given a set of fractions, the **least common denominator** is the smallest number that is divisible by each denominator.

Objective 3: Solving Linear Equations Involving Fractions

One way to solve a linear equation involving fractions is to first transform the equation into a linear equation involving integer coefficients. We can accomplish this using the multiplication property of equality by multiplying both sides of the equation by the lease common denominator (LCD).

Objective 4: Solving Linear Equations Involving Decimals

The strategy for solving linear equations involving decimals is similar to the one used to solve linear equations involving fractions. We want to eliminate all decimals by multiplying both sides of the equation by the smallest power of 10 (such as $10^1 = 10$, $10^2 = 100$, etc) that will guarantee that the new linear equation will not include decimals. To determine the smallest power of 10, look at all terms in the equation that contain a decimal factor and choose the factor that has the greatest number of decimal places. Count these decimal places and then raise 10 to that power.