Section 1.1b Rational Equations

Objective 5: Recognizing Rational Equations

A polynomial expression is an algebraic expression in which the exponents of all variable factors are nonnegative integers. For example, $3x^4 - 5x$, $-\frac{1}{2}x^2 - 5x + 3$, and 5 are polynomial expressions, but $x^{-2} + 3$, $4\sqrt[3]{x} - x^4$, and $\frac{3}{x} + 7x^5$ are not polynomial expressions.

A rational expression is one that can be written as the quotient of two polynomial expressions such that the numerator is any polynomial expression except 0 and the degree of the polynomial expression in the denominator is greater than or equal to one. (That is, there must be at least one variable in the polynomial expression in the denominator.)

Definition: A **rational equation** is an equation consisting of one or more rational expressions with any other expressions of the equation being polynomials.

For example,

$\frac{1}{x} = 10$ is a rational equation because $\frac{1}{x}$ is a rational expression and 10 is a polynomial.

$\frac{3x+5}{8} = \frac{9}{x^2-1}$ is a rational equation because $\frac{3x+5}{8}$ is a polynomial and $\frac{9}{x^2-1}$ is a rational expression.

However,

$\frac{1}{4}x + 2 = -6$ and $\frac{3x+5}{8} = x$ are not rational equations because they contain no rational expressions.

$\frac{1}{\sqrt{x}} + 5 = 9$ is not a rational equation because $\frac{1}{\sqrt{x}}$ is not a rational expression (since $\sqrt{x} = x^{\frac{1}{2}}$ is not a polynomial).

Review of Multiplying Two Binomials

See Section 1.4a. ***LSU Video "Multiplying Polynomials" (3:40 – 8:06)*** *is found on the course website.*

Review of Factoring Polynomials

See Section 1.4a for a list of LSU Videos available on the course website.

Review of Finding a Least Common Denominator for a Set of Rational Expressions

Given a set of rational expressions, the **least common denominator** is the smallest expression that is divisible by each denominator.

Objective 6: Solving Rational Equations That Lead to Linear Equations

The process of solving a rational equation is very similar to the process of solving linear equations containing fractions. That is, we first determine the least common denominator (LCD) and then multiply both sides of the equation by the LCD. We have to be extra cautious when solving rational equations because we have to be aware of the **restricted values**, that is, values that make the denominator of a rational expression to equal zero.

***Definition*:** An **extraneous solution** is a solution to an equation obtained through algebraic manipulations that is not a solution to the original equation.

Note: **Because rational equations often have an extraneous solution, it is imperative to first determine all restricted values before beginning the solution process.**

Solving Rational Equations

Step 1 Factor any denominators then list all restricted values.
Step 2 Determine the LCD of all denominators in the equation.
Step 3 Multiply both sides of the equation by the LCD.
Step 4 Solve the resulting equation.
Step 5 Discard any restricted values.