Section 1.4a Quadratic Equations

In Section 1.1 we studied linear equations of the form $ax+b=c,\ a\neq 0$. These equations are also known as 1st order polynomial equations. In section 1.4, we will learn how to solve 2nd order polynomial equations. Second order polynomial equations are called **quadratic equations**.

Definition: A **Quadratic Equation in One Variable** is an equation that can be written in the form $ax^2+bx+c=0,\quad a\neq 0$. Quadratic equations in this form are said to be in *standard form.*

Review of Multiplying Binomials
LSU Video "Multiplying Polynomials" (3:40 – 8:06) *is found on the course website.*

To multiply two binomials, we must use the distributive property. We saw in a previous section that one way to apply the distributive property to multiply two binomials is to use the FOIL method.

Review of Factoring a Greatest Common Factor.
LSU Video "Greatest Common Factor and Factoring by Grouping" (0:00 – 14:00) *is found on the course website.*

The first step to factoring a polynomial is to check to see if there is a **greatest common factor** (GCF) that can be factored out of each term. The GCF of a list of terms or monomials is the product of the GCF of the numerical coefficients and each GCF of the variable factors.

Review of Factoring by Grouping
LSU Video "Greatest Common Factor and Factoring by Grouping" (14:00 – 20:46) *is found on the course website*

Sometimes it is possible to factor a polynomial by grouping the terms of the polynomial and looking for common factors in each group. This method of factoring is called factoring by grouping. In particular, look to see if factoring by grouping will work when the polynomial has four terms.

Review of Factoring Trinomials of the Form $x^2 + bx + c$
LSU Video "Factoring Trinomials of the Form $x^2 + bx + c$" is *found on the course website.*

Consider the quadratic expression $x^2+3x-10$. Since $(x-2)(x+5)=x^2+3x-10$, we say that $(x-2)(x+5)$ is a **factored form** of $x^2+3x-10$.
The factored form of a quadratic expression is the product of two linear factors and possibly a constant. If a quadratic expression cannot be factored over the integers, then we say that it is **prime**.

Review of Factoring Trinomials of the Form $ax^2 + bx + c$
LSU Video " Factoring Trinomials of the Form $ax^2 + bx + c$" is *found on the course website.*

When the leading coefficient, a, is not equal to one, we will use one of two methods to factor the expression. The first is trial and error. Trial and error can be an efficient choice when a and c do not have many factor pairs.

Another method that can be used is factoring by grouping by first rewriting the trinomial as a four-term polynomial. This method is sometimes referred to as splitting the linear term or the ac method.

Steps For Factoring a Trinomial of the Form $ax^2 + bx + c$ by Grouping:

Step 1: Find two numbers that have a product of $a \cdot c$ and a sum of *b*.
Step 2: Write the term bx as a sum using the numbers found in Step 1.
Step 3: Factor by grouping.

Objective 1: Solving Quadratic Equations by Factoring and the Zero Product Property

Some quadratic equations can be easily **solved by factoring** and by using the following important property.

The Zero Product Property: If $AB = 0$ then $A = 0$ or $B = 0$.

The Zero Product Property says that if two factors multiplied together are equal to zero, then at least one of the factors must be zero.

Review of Factoring the Difference of Squares
***LSU Video "Factoring Binomials"* is found on the course website.**

A binomial is a **difference of two squares** when it is the difference of the square of some quantity a and the square of some quantity b.

Difference of two squares: $a^2 - b^2 = (a+b)(a-b)$

Review of Simplifying Square Roots

LSU Video "Simplifying Square Roots" *is found on the course website.*

A square root is simplified when the radicand contains no perfect square factors other than 1. For example, $\sqrt{20}$ is not simplified because $\sqrt{20}=\sqrt{4\cdot 5}$ and 4 is a perfect square.

Product Rule for Square Roots:
If $\sqrt{a}$ and $\sqrt{b}$ are real numbers, then $\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}$.

Applying this rule, we can simplify $\sqrt{20}$ as follows: $\sqrt{20}=\sqrt{4\cdot 5}=\sqrt{4}\cdot\sqrt{5}=2\sqrt{5}$

Objective 2: Solving Quadratic Equations using the Square Root Property

Any quadratic equation of the form $x^2-c=0$ where $c>0$ can be solved by factoring the left side as $\left(x-\sqrt{c}\right)\left(x+\sqrt{c}\right)=0$ thus the solutions are $x=\pm\sqrt{c}$. Quadratic equations of this form can be more readily solved by using the following **square root property**.

> **The Square Root Property:** The solution to the quadratic equation $x^2-c=0$, or equivalently $x^2=c$, is $x=\pm\sqrt{c}$.