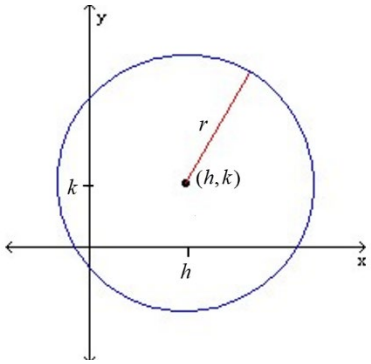
Section 2.2 Circles

A **circle** is the set of all points (x, y) in the Cartesian plane that are a fixed distance r from a fixed point (h,k). The fixed distance r is called the **radius** of the circle and the fixed point (h,k) is called the **center** of the circle. To derive the equation of a circle, we use the distance formula that was discussed in the previous section.



The **standard form of an equation of a circle** with center (h,k) and radius r is $(x-h)^2+(y-k)^2=r^2$.

The standard form of an equation of a circle centered at the origin with radius r is $x^2+y^2=r^2$.

Objective 1: Writing the Standard Form of an Equation of a Circle

Note that when given the diameter of a circle, we can use the midpoint formula to determine the center and the distance formula to find the radius.

Review of Solving Quadratic Equations by Using the Square Root Property
See Section 1.4a, objective 2.

Objective 2: Sketching the Graph of a Circle

Once we know the center and radius of a circle, we can easily graph the circle. For additional points, find any intercepts and plot the points.

Note that the y-coordinate of the center of the circle $(x-1)^2+(y+2)^2=9$ is $k=-2$ because $(y+2)^2=(y-(-2))^2$.

Review of Squaring Binomials
LSU Video "Special Products" (0:00 – 8:20) *is found on the course website.*

There are two identities that can be used to square a binomial.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

Review of Factoring Perfect Square Trinomials
LSU Video "Perfect Square Trinomials" *is found on the course website.*

A trinomial is a **perfect square trinomial** if it can be written so that its first term is the square of some quantity a, its last term is the square of some quantity b, and its middle term is twice the product of the quantities a and b.

The two identities from above that we used to square a binomial can also be used to factor a perfect square trinomial.

Review of Creating a Perfect Square Trinomial

We have previously used the square root property to solve quadratic equations such as $(x+1)^2 = 5$. Notice that one side of the equation is a quantity squared and the other side is a constant.

Consider the equation $x^2 + 2x = 4$. To solve this equation by using the square root property, we need the left side of the equation to be a perfect square trinomial, meaning it can be written as a binomial squared. We can do this by adding 1 to both sides of the equation.

$$x^2 + 2x = 4$$
$$x^2 + 2x + 1 = 4 + 1$$
$$(x+1)^2 = 5$$

The process of rewriting the equation so that one side is a perfect square trinomial is called **completing the square**.

Objective 3: Converting the General Form of a Circle into Standard Form

The **general form of the equation of a circle** is $Ax^2 + By^2 + Cx + Dy + E = 0$ where $A,\ B, C, D, \text{and } E$ are real numbers, $A = B$, $A \neq 0$, and $B \neq 0$.

By completing the square, the equation of a circle can be rewritten from general form to standard form.