

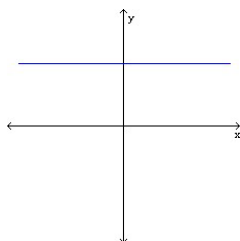
## Section 3.3 Graphs of Basic Functions; Piecewise Functions

### Objective 1: Sketching the Graphs of the Basic Functions

We begin by discussing the graphs of two specific linear functions. Recall that a linear function has the form  $f(x) = mx + b$  where  $m$  is the slope of the line and  $b$  represents the  $y$ -coordinate of the  $y$ -intercept.

We start our discussion of the basic functions by looking at the **constant function**, that is, the linear function with  $m = 0$ , the graph of which is a horizontal line.

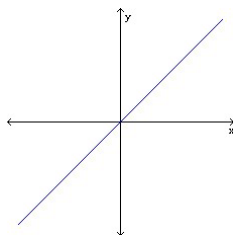
(1) The **Constant Function**  $f(x) = b$  has domain  $(-\infty, \infty)$  and range  $\{b\}$ .



*Notice that there are no arrows used at either end of the graph representing the constant function above. From this point forward in the text, unless the graph contains a definitive endpoint (shown by either an open dot or a closed dot) then it will be understood that the graph extends indefinitely in the same direction.*

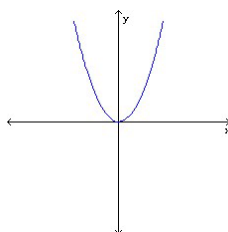
The **identity function** defined by  $f(x) = x$  is another linear function with  $m = 1$  and  $b = 0$ . It assigns to each number in the domain the exact same number in the range.

(2) The **Identity Function**  $f(x) = x$  has domain  $(-\infty, \infty)$  and range  $(-\infty, \infty)$ .



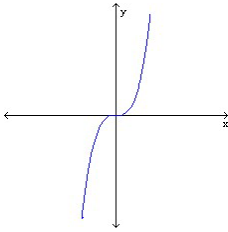
The **square function**,  $f(x) = x^2$ , assigns to each real number in the domain the square of that number in the range. The “u-shaped” graph of the square function is called a parabola.

(3) The **Square Function**  $f(x) = x^2$  has domain  $(-\infty, \infty)$  and range  $[0, \infty)$ .



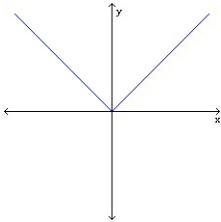
The **cube function**,  $f(x) = x^3$ , assigns to each real number in the domain the cube of that number in the range.

(4) The **Cube Function**  $f(x) = x^3$  has domain  $(-\infty, \infty)$  and range  $(-\infty, \infty)$ .



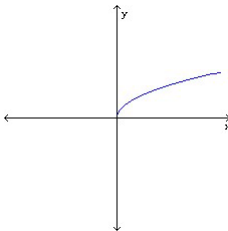
The **absolute value function**,  $f(x) = |x|$ , assigns to each real number in the domain the absolute value of that number in the range.

(5) The **Absolute Value Function**  $f(x) = |x|$  has domain  $(-\infty, \infty)$  and range  $[0, \infty)$ .



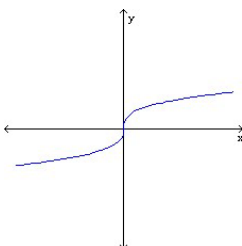
The **square root function**,  $f(x) = \sqrt{x}$ , is only defined for values of  $x$  that are greater than or equal to zero. It assigns to each real number in the domain the square root of that number in the range.

(6) The **Square Root Function**  $f(x) = \sqrt{x}$  has domain  $[0, \infty)$  and range  $[0, \infty)$ .



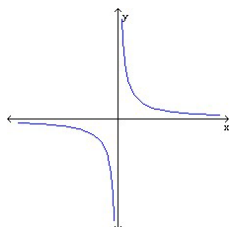
Unlike the square root function which is only defined for values of  $x$  greater than or equal to zero, the **cube root function**,  $f(x) = \sqrt[3]{x}$ , is defined for all real numbers and assigns to each number in the domain the cube root of that number in the range.

(7) The **Cube Root Function**  $f(x) = \sqrt[3]{x}$  has domain  $(-\infty, \infty)$  and range  $(-\infty, \infty)$ .



The **reciprocal function**,  $f(x) = \frac{1}{x}$ , is a rational function whose domain is  $\{x | x \neq 0\}$ . It assigns to each number  $a$  in the domain its reciprocal,  $\frac{1}{a}$ , in the range. The reciprocal function has two asymptotes. The  $y$ -axis (the line  $x = 0$ ) is a vertical asymptote and the  $x$ -axis (the line  $y = 0$ ) is a horizontal asymptote.

(8) The **Reciprocal Function**  $f(x) = \frac{1}{x}$  has domain  $(-\infty, 0) \cup (0, \infty)$  and range  $(-\infty, 0) \cup (0, \infty)$ .

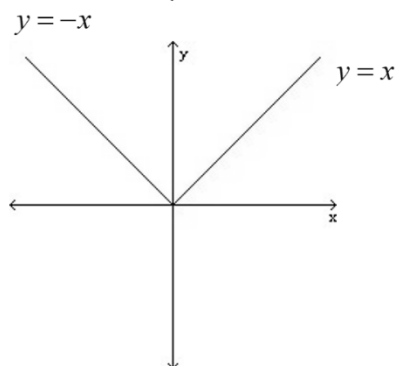


## Objective 2: Sketching the Graphs of Basic Functions with Restricted Domains

## Objective 3: Analyzing Piecewise Defined Functions

The absolute value function,  $f(x) = |x|$ , can also be defined by a rule that has two different “pieces.”

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



You can see by the graph above that the “left-hand piece” is actually a part of the line  $y = -x$  while the “right-hand piece” is a part of the line  $y = x$ .

Functions defined by a rule that has more than one “piece” are called piecewise-defined functions.

