# Section 3.3 Graphs of Basic Functions; Piecewise Functions

Objective 1: Sketching the Graphs of the Basic Functions

We begin by discussing the graphs of two specific linear functions. Recall that a linear function has the form $f(x) = mx + b$ where m is the slope of the line and b represents the y-coordinate of the y-intercept.

We start our discussion of the basic functions by looking at the **constant function**, that is, the linear function with $m = 0$, the graph of which is a horizontal line.

(1) The **Constant Function** $f(x) = b$ has domain $(-\infty, \infty)$ and range $\{b\}$.
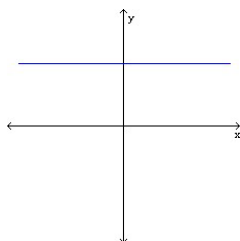
Notice that there are no arrows used at either end of the graph representing the constant function above. From this point forward in the text, unless the graph contains a definitive endpoint (shown by either an open dot or a closed dot) then it will be understood that the graph extends indefinitely in the same direction.

The **identity function** defined by $f(x) = x$ is another linear function with $m = 1$ and $b = 0$. It assigns to each number in the domain the exact same number in the range.

(2) The **Identity Function** $f(x) = x$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$.
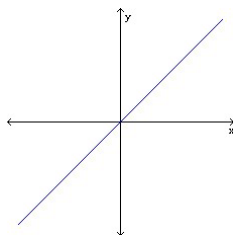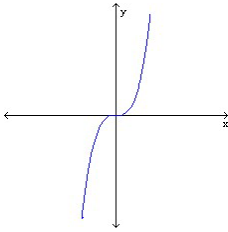

The **square function,** $f(x) = x^2$, assigns to each real number in the domain the square of that number in the range. The "u-shaped" graph of the square function is called a parabola.

(3) The **Square Function** $f(x) = x^2$ has domain $(-\infty, \infty)$ and range $[0, \infty)$.
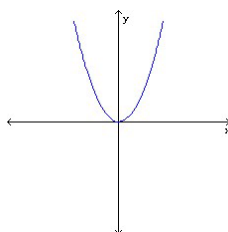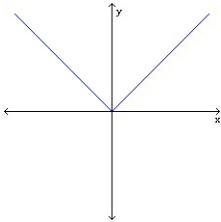
The **cube function**, $f(x) = x^3$, assigns to each real number in the domain the cube of that number in the range.

(4) The **Cube Function** $f(x) = x^3$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$.

The **absolute value function**, $f(x) = |x|$, assigns to each real number in the domain the absolute value of that number in the range.

(5) The **Absolute Value Function** $f(x) = |x|$ has domain $(-\infty, \infty)$ and range $[0, \infty)$.

The **square root function**, $f(x) = \sqrt{x}$, is only defined for values of x that are greater than or equal to zero. It assigns to each real number in the domain the square root of that number in the range.

(6) The **Square Root Function** $f(x) = \sqrt{x}$ has domain $[0, \infty)$ and range $[0, \infty)$.
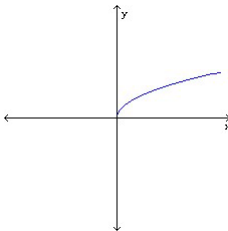
Unlike the square root function which is only defined for values of x greater than or equal to zero, the **cube root function,** $f(x) = \sqrt[3]{x}$, is defined for all real numbers and assigns to each number in the domain the cube root of that number in the range.

(7) The **Cube Root Function** $f(x) = \sqrt[3]{x}$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$.
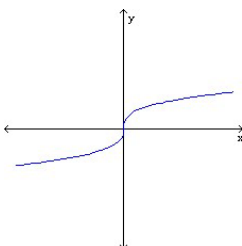
The **reciprocal function**, $f(x) = \dfrac{1}{x}$, is a rational function whose domain is $\{x \mid x \neq 0\}$. It assigns to each number a in the domain its reciprocal, $\dfrac{1}{a}$, in the range. The reciprocal function has two asymptotes. The y-axis (the line $x = 0$) is a vertical asymptote and the x-axis (the line $y = 0$) is a horizontal asymptote.

(8) The **Reciprocal Function** $f(x) = \dfrac{1}{x}$ has domain $(-\infty, 0) \cup (0, \infty)$ and range $(-\infty, 0) \cup (0, \infty)$.
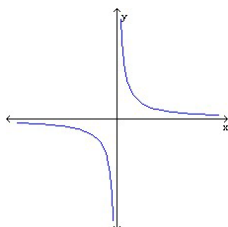
Objective 2: Sketching the Graphs of Basic Functions with Restricted Domains

Review of Functions: Using the Vertical Line Test and Function Notation
*LSU Video **"Functions" (6:43 – 11:35 and 13:45 – 20:38)** is available on the course website.*
See also Section 3.1.

Review of Determining the Domain and Range of a Function from its Graph
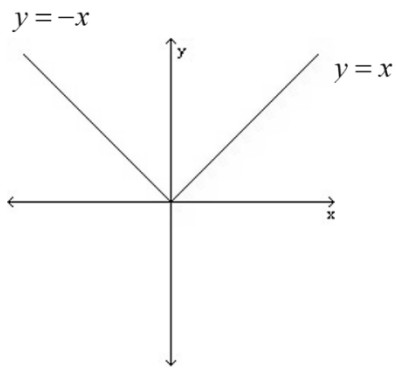See Section 3.2.

Review of Sketching the Graphs of Linear Functions
A **linear function** has the form $f(x) = mx + b$ where m is the slope of the line and b is the y-intercept.

Objective 3: Analyzing Piecewise Defined Functions

The absolute value function, $f(x) = |x|$, can also be defined by a rule that has two different "pieces."

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$y = -x$

$y = x$

You can see by the graph above that the "left-hand piece" is actually a part of the line $y = -x$ while the "right-hand piece" is a part of the line $y = x$.

Functions defined by a rule that has more than one "piece" are called piecewise-defined functions.