Section 3.4 Transformations of Functions

Review of Sketching the Graphs of the Basic Functions and Sketching the Graphs of the Basic Functions with Restricted Domains

See Section 3.3.

Objective 1: Using Vertical Shifts to Graph Functions

Let c be a positive real number.

1. The graph of $y = f(x) + c$ is obtained by shifting the graph of $y = f(x)$ vertically upward c units.
2. The graph of $y = f(x) - c$ is obtained by shifting the graph of $y = f(x)$ vertically downward c units.

Objective 2: Using Horizontal Shifts to Graph Functions

Let c be a positive real number.

1. The graph of $y = f(x + c)$ is obtained by shifting the graph of $y = f(x)$ horizontally to the left c units.
2. The graph of $y = f(x - c)$ is obtained by shifting the graph of $y = f(x)$ horizontally to the right c units.

For $c > 0$, the graph of $y = f(x - c)$ is the graph of *f* shifted to the **right** c units. At first glance, it appears that the rule for horizontal shifts is the opposite of what seems natural. Substituting $x + c$ for x causes the graph of $y = f(x)$ to be shifted to the left while substituting $x - c$ for x causes the graph to shift to the right *c* units.

Objective 3: Using Reflections to Graph Functions

The graph of $y = -f(x)$ is a **reflection** of the graph of $y = f(x)$ **about the *x*-axis**.

The graph of $y = f(-x)$ is a **reflection** of the graph of $y = f(x)$ **about the *y*-axis**.

Objective 4: Using Vertical Stretches and Compressions to Graph Functions

Suppose a is a positive real number. The graph of $y = af(x)$ is obtained by the multiplying each y-coordinate of $y = f(x)$ by a.

1. If $a > 1$, the graph of $y = af(x)$ is a **vertical stretch** of the graph of $y = f(x)$.
2. If $0 < a < 1$, the graph of $y = af(x)$ is a **vertical compression** of the graph of $y = f(x)$.

Objective 6: Using Combinations of Transformations to Graph Functions

When graphing a function that involves multiple transformations, it is important to follow a certain "order of operations." In our text, transformations are performed in the following order:

1. Horizontal shifts
2. Reflection about y-axis
3. Vertical stretches/compressions
4. Reflection about x-axis
5. Vertical shifts

Although different ordering is possible, the order above will always work.

Objective 7: Using Transformations to Sketch the Graphs of Piecewise-Defined Functions