

Section 4.1 Quadratic Functions

In section 3.3, we learned the graphs of eight basic functions, including the square function $f(x) = x^2$. The square function is the simplest quadratic function.

If $x = -3$, then $y = (-3)^2$, or 9.

If $x = -2$, then $y = (-2)^2$, or 4.

If $x = -1$, then $y = (-1)^2$, or 1.

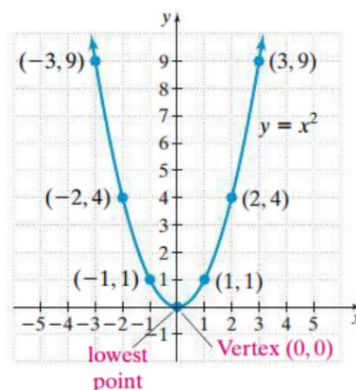
If $x = 0$, then $y = 0^2$, or 0.

If $x = 1$, then $y = 1^2$, or 1.

If $x = 2$, then $y = 2^2$, or 4.

If $x = 3$, then $y = 3^2$, or 9.

| $y = x^2$ | |
|-----------|-----|
| x | y |
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |



Review of Graphing Transformations of the Square Function

LSU Video “Quadratic Functions and Their Graphs” (0:00 – 20:35) is found on the course website.

See also Section 3.4.

Review of Solving Quadratic Equations

Recall that in section 1.4, we learned three methods of solving a quadratic equation: factoring, using the square root property, and using the quadratic formula.

Review of Evaluating Functions for Given Inputs

See Section 3.1.

Objective 1: Understanding the Definition of a Quadratic Function and its Graph

Definition: A **quadratic function** is a function that can be written in the form $f(x) = ax^2 + bx + c$ where a , b , and c are real numbers with $a \neq 0$. Every quadratic function has a “u-shaped” graph called a *parabola*.

The five basic characteristics of a parabola are its

1. vertex
2. axis of symmetry
3. y -intercept
4. x -intercept(s) or real zeros, and
5. domain and range.

The domain of a quadratic function is $(-\infty, \infty)$.

The parabola *opens up* if $a > 0$, so the function has a minimum value at the vertex. That minimum value is the y -coordinate of the vertex.

The parabola *opens down* if $a < 0$, so the function has a maximum value at the vertex. That maximum value is the y -coordinate of the vertex.

The x -intercept(s), if any, are found by solving the equation $f(x) = 0$. The y -intercept is $f(0)$.

Objective 2: Graphing Quadratic Functions Written in Vertex Form

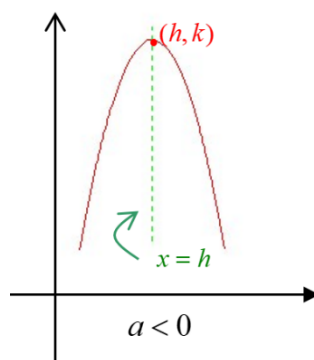
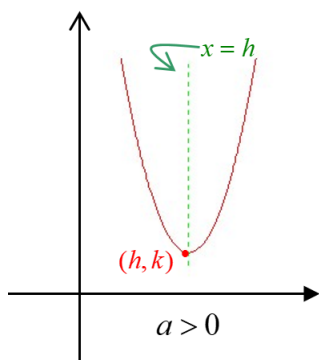
Vertex Form of a Quadratic Function

A quadratic function is in **vertex form** if it is written as $f(x) = a(x-h)^2 + k$.

The vertex of the parabola is (h, k) .

The line $x = h$ is the axis of symmetry.

The range is $[k, \infty)$ if $a > 0$, and the range is $(-\infty, k]$ if $a < 0$.



Objective 4: Graphing Quadratic Functions Using the Vertex Formula

Formula for the Vertex of a Parabola

Given a quadratic function of the form $f(x) = ax^2 + bx + c$, $a \neq 0$, the vertex of the parabola is

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

Objective 5: Determining the Equation of a Quadratic Function Given its Graph