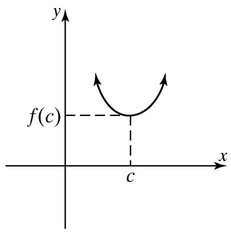
Section 4.2 Applications and Modeling of Quadratic Functions

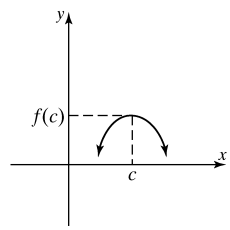
Review of Determining Relative Maximum and Relative Minimum Values of a Function
In section 3.2, we learned how to identify relative maximum and minimum values of a function when given its graph.

When a function $f(x)$ changes from increasing to decreasing at a point $(c, f(c))$, then f is said to have a relative maximum at $x = c$. The relative maximum value is $f(c)$.

When a function $f(x)$ changes from decreasing to increasing at a point $(c, f(c))$, then f is said to have a relative minimum at $x = c$. The relative minimum value is $f(c)$.



The relative minimum occurs at $x = c$, the relative minimum value is $f(c)$.

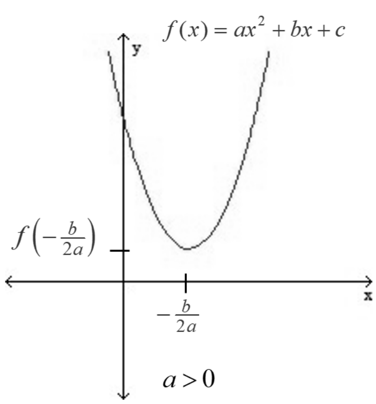


The relative maximum occurs at $x = c$, the relative maximum value is $f(c)$.

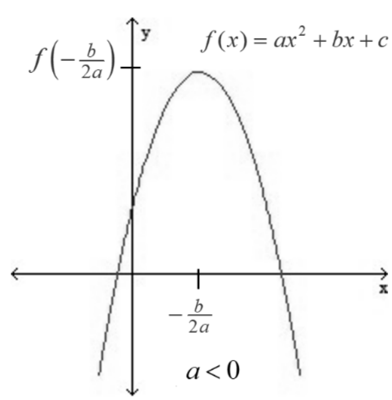
With application problems involving functions, we are often interested in finding the maximum or minimum value of the function. For example, a professional golfer may want to control the trajectory of a struck golf ball. Thus, she may be interested in determining the maximum height of the ball given certain parameters such as swing velocity and club angle. An economist may want to minimize a cost function or maximize a revenue or profit function. A builder with a fixed amount of fencing may want to maximize an area function.

In a calculus course, you will learn how to maximize or minimize a wide variety of functions. In this section, we will concentrate only on quadratic functions. Quadratic functions are relatively easy to maximize or minimize because we know a formula for finding the coordinates of the vertex. Recall that if $f(x) = ax^2 + bx + c,\ a \neq 0$, we know that the coordinates of the vertex are $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$.

If $a > 0$, the parabola *opens up* and has a minimum value at the vertex. The minimum value is the y-coordinate of the vertex.
If $a < 0$, the parabola *opens down* and has a maximum value at the vertex. The maximum value is the y-coordinate of the vertex.



Minimum value at vertex



Maximum value at vertex

Objective 1 Maximizing Projectile Motion Functions

An object launched, thrown, or shot vertically into the air with an initial velocity of v_0 meters/second (m/s) from an initial height of h_0 meters above the ground can be modeled by the function $h(t) = -4.9t^2 + v_0 t + h_0$ where $h(t)$ is the height of the projectile t seconds after its departure*.

*Note that the leading coefficient of the projectile motion model is -4.9. This constant is derived using calculus and the acceleration of gravity on earth, which is 9.8 meters per second per second. If the height was measured in feet, the leading coefficient of the projectile motion model would be -16 derived using calculus and acceleration of gravity on earth, which is 32 feet per second per second.

Review of Adding and Subtracting Polynomials; Multiplying a Polynomial by a Monomial
See Section R.5 in the etext.

Objective 2 Maximizing Functions in Economics

Revenue is defined as the dollar amount received by selling x items at a price of p dollars per item, that is, $R = xp$. For example, if a child sells 50 cups of lemonade at a price of $0.25 per cup, then the revenue generated is $R = xp = (50)(0.25) = \$12.50$.

A common model in economics, The Law of Supply and Demand, states that as the quantity, x, increases, the price, p, tends to decrease. Likewise, if the quantity decreases, the price tends to increase. Equations (or functions) that relate the quantity, x, and the price, p, are called **demand equations** or demand functions.

Profit is equal to revenue minus cost.