Section 8.6 Compositions of Reflections

# Objective 1: Find Compositions of Transformations, Including Glide Reflections

Any isometry can be expressed as a composition of reflections. If two figures in a plane are congruent, we can map one onto the other using a composition of reflections.

**Theorem**

A translation or rotation is a composition of two reflections.

**Theorem**

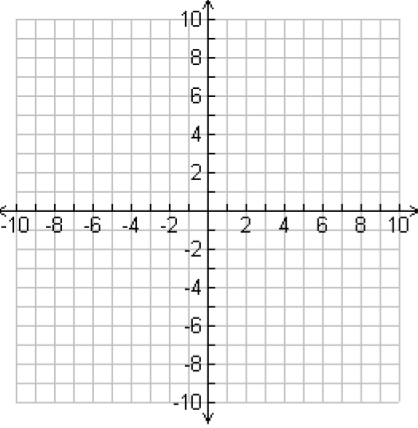
* A composition of reflections across two parallel lines is a translation.
* A composition of reflections across two intersecting lines is a rotation.

Shape

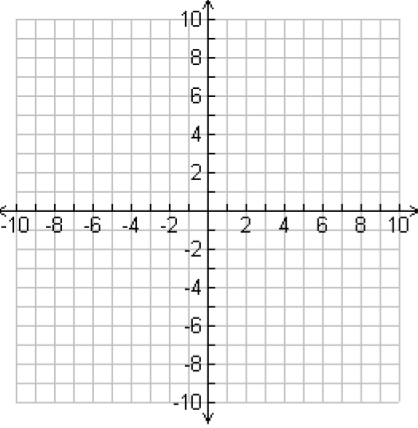
Description automatically generated

a. Use the given points and lines to graph  and its image after a reflection first across line *L*1 and then across line *L*2. Is the resulting transformation a translation or a rotation? For a translation, describe the direction and distance. For a rotation, state the center of rotation and the angle of rotation.

i. , ; , 



ii. , ; *L*1: *x*-axis, *L*2: *y*-axis

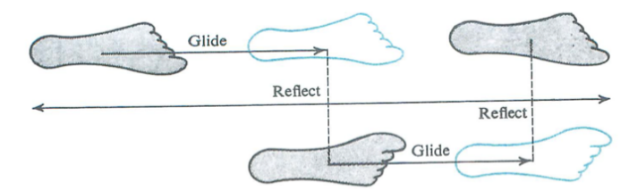


Suppose two figures *A* and *B* in a plane are congruent but have opposite orientations. Then, the reflection *A*’ of *A* has the same orientation as *B*. *B* is a translation or rotation image of *A*’, so therefore, *A*’ can be mapped to *B* using two reflections. This means *A* can be mapped to *B* using at most three reflections. This is an important theorem.

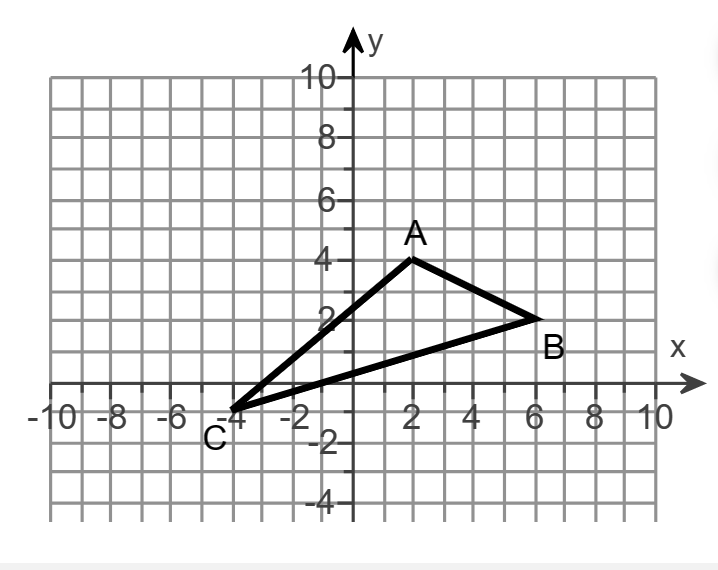
**Theorem: Fundamental Theorem of Isometries**

In a plane, one of two congruent figures can be mapped to the other by a composition of at most three reflections.

If two figures are congruent and have opposite orientations (but are not simply reflections of each other, there are a translation and a reflection that will map one onto the other. A **glide reflection** is the composition of a translation (or glide) and a reflection across a line parallel to the direction of the translation. Footprints are a real-life example of a glide reflection.



b. Find the glide reflection image of the triangle shown below given the translation  and the reflection line.



c. Find the coordinates of point P under the transformation  for  and the reflection line.

# Objective 2: Classify Isometries

**Theorem: Isometry Classification Theorem**

There are only four isometries: translations, rotations, reflections, and glide reflections. In translations and rotations, the orientation remains the same.

Graphical user interface, application

Description automatically generated

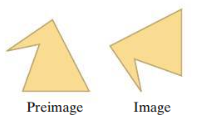
In reflections and glide reflections, the orientation is opposite.

A picture containing text

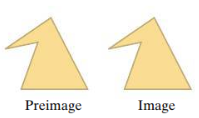
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a. Each transformation is an isometry. Classify the isometry as a translation, reflection, rotation, or glide reflection.

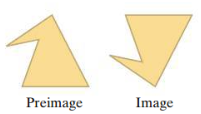
i.



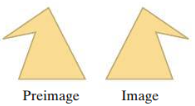
ii.



iii.



iv.



b. Identify each mapping as a single translation, reflection, rotation, or glide reflection. Find the translation rule, reflection line, center of rotation and angle of rotation, or glide translation rule and reflection line.

point A has coordinates negative 9, 0.
point B has coordinates negative 9, negative 4. point C has coordinates negative 7, 0. point D has coordinates negiative 5, 4. point E has coordinates negative 5, 0. point F has coordinates negative 4, 0. point G has coordinates negative 2, 4. point H has coordinates negative 2, 0. poing I has coordinates negative 2, negative 4.  point J has coordinates 2, 0. point L has coordinates 2, negative 4. point M has coordinates 4, 0.  point N has coordinates 4, 4. Point Q has coordinates 6, negative 4.  Point P has coordinates 6, 0.

i. 

ii. 

iii. 

iv. 