Section 1.1   Linear and Rational Equations

Objective 1: Recognizing Linear Equations

A linear equation in one variable involves variables that are only raised to the first power.

Definition: A **linear equation in one variable** is an equation that can be written in the form $ax + b = c$, where a, b, and c are real numbers and $a \neq 0$.

Examples of linear equations:

$$\frac{7}{2}x - 4 = x \qquad \sqrt{5}x - 3 = 8x - 1 \qquad (0.7)^2 x + 1 = 6 \qquad 2^{-1} - 0.4x = 0.3 - 5x$$

Examples of non-linear equations:

$$\frac{3}{x} - 5 = 9 \qquad 5\sqrt{x} + 1 = 7 \qquad x^2 + x = 6 \qquad (0.7x)^2 + x = 1 \qquad 0.3 - 0.2x^{-1} = 2$$

Objective 2: Solving Linear Equations with Integer Coefficients

When we solve an equation for *x*, we are looking for all values of *x* which, when substituted back into the original equation, yield a true statement. The goal here is to **isolate the variable** *x* on one side of the equation.

Objective 3: Solving Linear Equations Involving Fractions

Objective 4: Solving Linear Equations Involving Decimals

The strategy for solving linear equations involving decimals is similar to the one used to solve linear equations involving fractions. We want to eliminate all decimals.

Objective 5: Recognizing Rational Equations

A rational expression is the quotient of two polynomials such that the degree of the polynomial in the denominator is greater than or equal to one.

Definition: A **Rational Equation** is an equation consisting of one or more rational expressions with any other expressions of the equation being polynomials.

Examples of rational equations:

$$\frac{1}{x} = 10 \qquad\qquad \frac{3x+5}{8} = \frac{9}{x^2-1} \qquad\qquad \frac{3x+5}{8x} = x$$

Examples of non-rational equations:

$$\frac{1}{4}x + 2 = -6 \qquad\qquad \frac{1}{\sqrt{x}} + 5 = 9 \qquad\qquad \frac{3x+5}{8} = x$$

Objective 6: Solving Rational Equations That Lead to Linear Equations

The process of solving a rational equation is very similar to the process of solving linear equations containing fractions. That is, we first determine the least common denominator (LCD) and then multiply both sides of the equation by the LCD. We have to be extra cautious when solving rational equations because we have to be aware of the restricted values, that is, values that make the denominator of a rational expression to equal zero.

Definition: An **extraneous solution** is a solution to an equation obtained through algebraic manipulations that is not a solution to the original equation.

Note: Because rational equations often have an extraneous solution, it is imperative to first determine all restricted values before beginning the solution process.

Solving Rational Equations

Step 1 Factor any denominators then list all restricted values.
Step 2 Determine the LCD of all denominators in the equation.
Step 3 Multiply both sides of the equation by the LCD.
Step 4 Solve the resulting equation.
Step 5 Discard any restricted values.