Section 1.4 Quadratic Equations

In Section 1.1 we studied linear equations of the form $ax + b = c$, $a \neq 0$. These equations are also known as 1st order polynomial equations. In this section, we will learn how to solve 2nd order polynomial equations. Second order polynomial equations are called **quadratic equations**.

**Definition**: A **Quadratic Equation in One Variable** is an equation that can be written in the form $ax^2 + bx + c = 0$, $a \neq 0$. Quadratic equations in this form are said to be in **standard form**.

**Objective 1: Solving Quadratic Equations by Factoring and the Zero Product Property**

Some quadratic equations can be easily **solved by factoring** and by using the following important property.

**The Zero Product Property**: If $AB = 0$ then $A = 0$ or $B = 0$.

The zero product property says that if two factors multiplied together are equal to zero, then at least one of the factors must be zero.

**Objective 2: Solving Quadratic Equations using the Square Root Property**

Any quadratic equation of the form $x^2 - c = 0$ where $c > 0$ can be solved by factoring the left side as $(x - \sqrt{c})(x + \sqrt{c}) = 0$ thus the solutions are $x = \pm \sqrt{c}$. Quadratic equations of this form can be more readily solved by using the following **square root property**.

**The Square Root Property**: The solution to the quadratic equation $x^2 - c = 0$, or equivalently $x^2 = c$, is $x = \pm \sqrt{c}$. 
Objective 4: Solving Quadratic Equations Using the Quadratic Formula

The **quadratic formula** can be used to solve any quadratic equation.

The **Quadratic Formula**: The solution to the quadratic equation \( ax^2 + bx + c = 0, \ a \neq 0 \) is given by the formula \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

Objective 5: Using the Discriminant to Determine the Type of Solutions of a Quadratic Equation

Given a quadratic equation of the form \( ax^2 + bx + c = 0, \ a \neq 0 \), the expression \( b^2 - 4ac \) is called the **discriminant**. Knowing the value of the discriminate can help us determine the number and nature of the solutions to a quadratic equation.

**The Discriminant**: Given a quadratic equation \( ax^2 + bx + c = 0, \ a \neq 0 \), the expression \( D = b^2 - 4ac \) is called the **discriminant**.

If \( D > 0 \), then the quadratic equation has two real solutions.
If \( D < 0 \), then the quadratic equation has two non-real solutions.
If \( D = 0 \), then the quadratic equation has exactly one real solution.