Section 1.4   Quadratic Equations

In Section 1.1 we studied linear equations of the form $ax + b = c$, $a \neq 0$. These equations are also known as 1st order polynomial equations. In this section, we will learn how to solve 2nd order polynomial equations. Second order polynomial equations are called **quadratic equations**.

Definition: A **Quadratic Equation in One Variable** is an equation that can be written in the form $ax^2 + bx + c = 0$, $a \neq 0$. Quadratic equations in this form are said to be in *standard form.*

Objective 1: Solving Quadratic Equations by Factoring and the Zero Product Property

Some quadratic equations can be easily **solved by factoring** and by using the following important property.

> **The Zero Product Property:** If $AB = 0$ then $A = 0$ or $B = 0$.

The zero product property says that if two factors multiplied together are equal to zero, then at least one of the factors must be zero.

Objective 2: Solving Quadratic Equations using the Square Root Property

Any quadratic equation of the form $x^2 - c = 0$ where $c > 0$ can be solved by factoring the left side as $\left(x - \sqrt{c}\right)\left(x + \sqrt{c}\right) = 0$ thus the solutions are $x = \pm\sqrt{c}$. Quadratic equations of this form can be more readily solved by using the following **square root property**.

> **The Square Root Property:** The solution to the quadratic equation $x^2 - c = 0$, or equivalently $x^2 = c$, is $x = \pm\sqrt{c}$.

Objective 4: Solving Quadratic Equations Using the Quadratic Formula

The **quadratic formula** can be used to solve any quadratic equation.

> **The Quadratic Formula:** The solution to the quadratic equation $ax^2 + bx + c = 0, \ a \neq 0$ is given by the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Objective 5: Using the Discriminant to Determine the Type of Solutions of a Quadratic Equation

Given a quadratic equation of the form $ax^2 + bx + c = 0, \ a \neq 0$, the expression $b^2 - 4ac$ is called the **discriminant**. Knowing the value of the discriminate can help us determine the number and nature of the solutions to a quadratic equation.

> **The Discriminant:** Given a quadratic equation $ax^2 + bx + c = 0, \ a \neq 0$, the expression $D = b^2 - 4ac$ is called the **discriminant**.
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> If $D > 0$, then the quadratic equation has two real solutions.
> If $D < 0$, then the quadratic equation has two non-real solutions.
> If $D = 0$, then the quadratic equation has exactly one real solution.