Section 1.6 Other Types of Equations

Objective 1: Solving Higher Order Polynomial Equations

So far in this text we have learned methods for solving linear equations and quadratic equations. Linear equations and quadratic equations are both examples of polynomial equations of first and second degree, respectively. In this section we will first start by looking at certain higher order polynomial equations that can be solved using special factoring techniques.

One useful technique is to factor out an expression included in each term, or **remove the common factor**, and then use the zero product property.

In an equation of the form $ax^3 + bx^2 + cx = 0$ above, do <u>not</u> divide both sides by *x*. This would produce the equation $ax^2 + bx + c = 0$, which has only two solutions. The solution $x = 0$ would be "lost." In addition, because $x = 0$ is a solution of the original equation, dividing by *x* would mean dividing by 0, which of course is undefined and produces incorrect results.

Sometimes polynomials can be solved by grouping terms and factoring (especially when the polynomial has four terms), or **factor by grouping**. Arrange the terms of the polynomial in descending order and group the terms of the polynomial in pairs.

Objective 2: Solving Equations that are Quadratic in Form ("Disguised Quadratics")

Quadratic equations of the form $ax^2+bx+c=0,\ a\neq 0$ are relatively straight-forward to solve since we know several methods for solving quadratics. Sometimes equations that are not quadratic can be made into a quadratic equation by using a **substitution**. Equations of this type are said to be *quadratic in form* or *"disguised quadratics"*. These equations typically have the form $au^2+bu+c=0, a\neq 0$ after an appropriate substitution.

Original Equation	**Identify *u*.**	**Find u^2.**	**Make the substitutions.**
$2x^4-11x^2+12=0$	$u=x^2$	$u^2=(x^2)^2=x^4$	$2u^2-11u+12=0$
$\left(\frac{1}{x-2}\right)^2+\frac{3}{x-2}-15=0$	$u=\frac{1}{x-2}$	$u^2=\left(\frac{1}{x-2}\right)^2$	$u^2+3u-15=0$
$x^{2/3}-9x^{1/3}+8=0$	$u=x^{1/3}$	$u^2=(x^{1/3})^2=x^{2/3}$	$u^2-9u+8=0$
$3x^{-2}-5x^{-1}-2=0$	$u=x^{-1}$	$u^2=(x^{-1})^2=x^{-2}$	$3u^2-5u-2=0$

Objective 3: Solving Equations Involving Radicals

A radical equation is an equation that involves a variable inside a square root, cube root or any higher root. To solve these equations we must try to isolate the radical, and then raise each side of the equation to the appropriate power to **eliminate the radical**.

Because the "squaring operation" can make a false statement true, ($-2 \neq 2$ but $(-2)^2 = (2)^2$**, for example), it is essential to always check your answers after solving an equation in which this operation was performed.**

Be careful when squaring an expression of the form $\boldsymbol{(a+b)^2}$ **or** $\boldsymbol{(a-b)^2}$**. Remember,**

$\boldsymbol{(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2}$ **and** $\boldsymbol{(a-b)^2 = (a-b)(a-b) = a^2 - 2ab + b^2}$ **.**