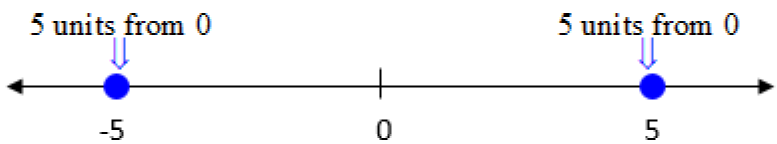
Section 1.8 Absolute Value Equations and Inequalities

When solving an absolute value equation or inequality, it is necessary to first isolate the absolute value expression.

Objective 1: Solving an Absolute Value Equation

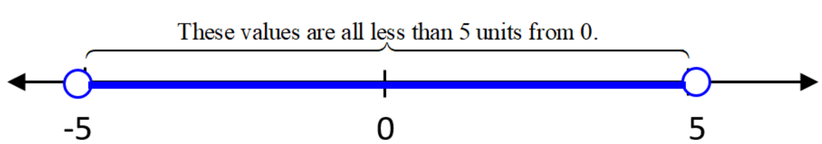
The absolute value of a number *x*, written as $|x|$, represents the **distance** from a number *x* to 0 on the number line. Consider the equation $|x| = 5$. To solve for *x*, we must find all values of *x* that are 5 units away from 0 on the number line. The two numbers that are 5 units away from 0 on the number line are $x = -5$ and $x = 5$ as shown in the figure below. Therefore, the solution set for $|x| = 5$ is $\{-5, 5\}$.



Objective 2: Solving Absolute Value Inequalities

Solving an Absolute Value "Less Than" Inequality

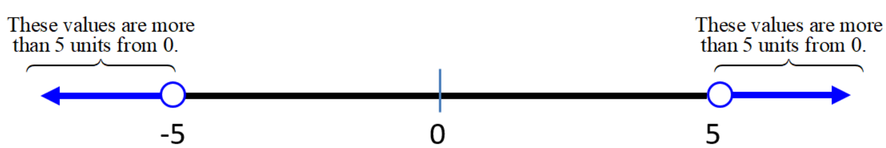
The solution to the inequality $|x| < 5$ consists of all values of *x* whose distance from 0 is less than 5 units on the number line. See the figure below.



If $|x| < 5$, then $-5 < x < 5$. The solution set is $\{x \mid -5 < x < 5\}$ in set builder notation or $(-5, 5)$ in interval notation.

Solving an Absolute Value "Greater Than" Inequality

For the solution to the inequality $|x| > 5$, notice that we are now looking for all values of *x* that are more than 5 units away from 0. The solution is the set of all values of *x* greater than 5 combined with the set of all values of *x* less than -5. See the figure below.



If $|x| > 5$, then $x < -5$ or $x > 5$. The solution set is $\{x \mid x < -5 \text{ or } x > 5\}$ in set builder notation or $(-\infty, -5) \cup (5, \infty)$ in interval notation.

$|5x+1|>3$ is NOT equivalent to $-3>5x+1>3$. In addition, a common error on this type of problem is to write $5x+1>-3$ for the first inequality instead of $5x+1<-3$. Think carefully about the meaning of the inequality before writing it.

ABSOLUTE VALUE EQUATIONS AND INEQUALITY PROPERTIES

Let *u* be an algebraic expression and let *c* be a real number such that $c>0$, then:

1. $|u|=c$ is equivalent to $u=-c$ or $u=c$.
2. $|u|<c$ is equivalent to $-c<u<c$.
3. $|u|>c$ is equivalent to $u<-c$ or $u>c$.