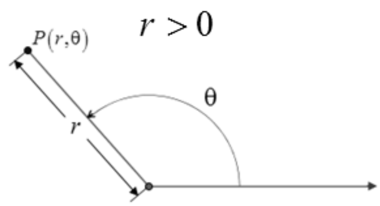
10.1 Polar Coordinates and Polar Equations

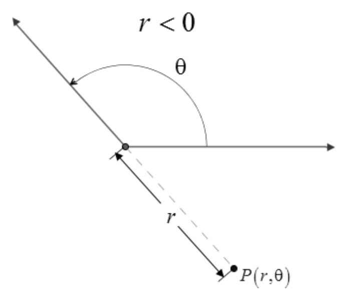
OBJECTIVE 1: Plotting Points Using Polar Coordinates

Given an ordered pair $P(r,\theta)$ in the polar coordinate system, the **directed distance** *r* can be positive, negative, or zero.

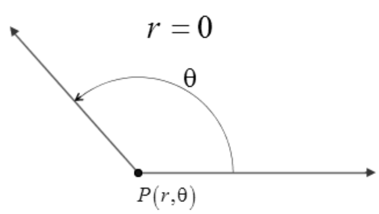
- If $r>0$, then *P* lies on the terminal side of angle θ.



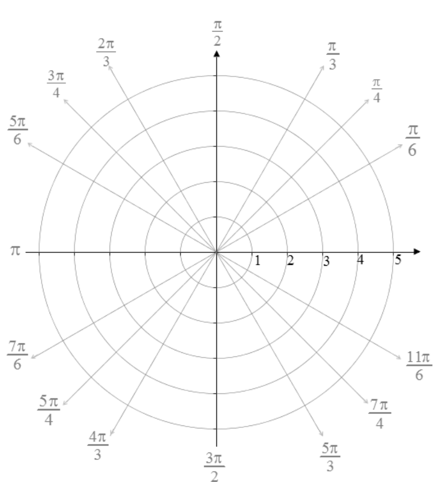
- If $r<0$, then *P* lies on the ray opposite of the terminal side of angle θ.



- If $r=0$, then *P* lies on the pole regardless of the measure of angle θ.

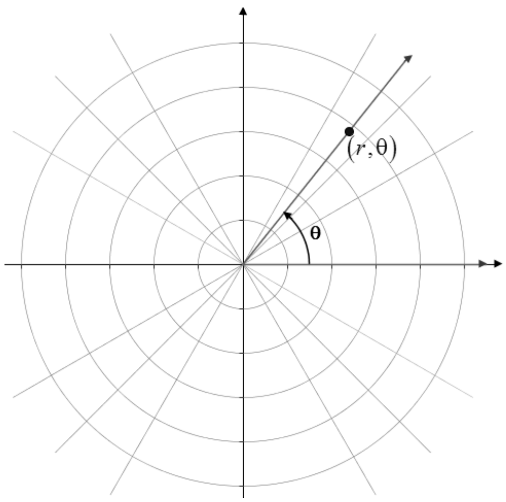


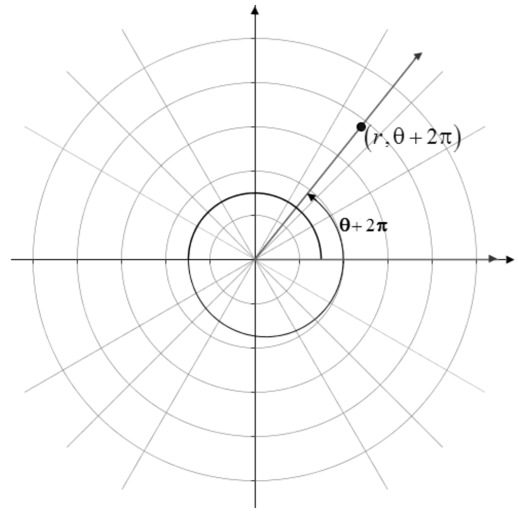
When plotting polar coordinates and sketching polar equations, we will often use a **polar grid**. A polar grid consists of a series of concentric circles of different radii and pre-sketched angles in standard position. Polar grid paper is available for free online if you wish to print and use it.



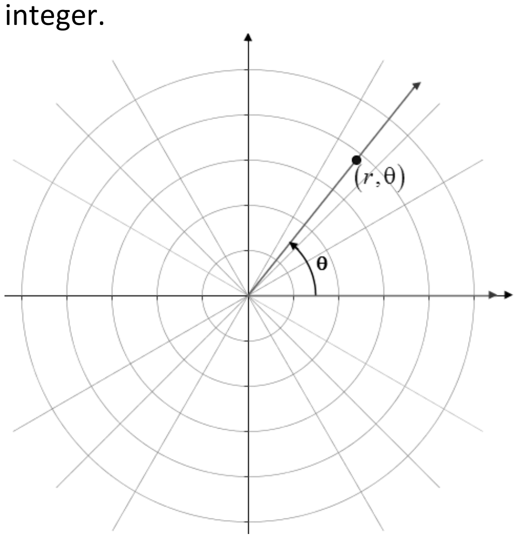
OBJECTIVE 2: Determining Different Representations of the Point (r, θ)

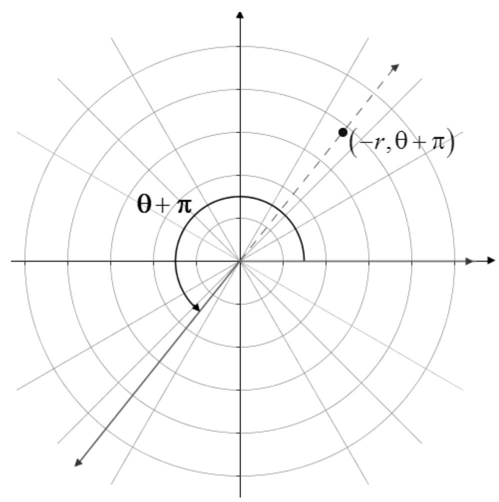
- Use the same value of *r* but choose an angle coterminal to θ. The coordinates will be of the form $(r, \theta + 2\pi k)$ where *k* is any integer.





- Use the opposite value of *r* but choose an angle coterminal to the angle located one-half of a rotation from angle θ. The coordinates will be of the form $(-r, \theta + \pi + 2\pi k)$ where *k* is any integer.





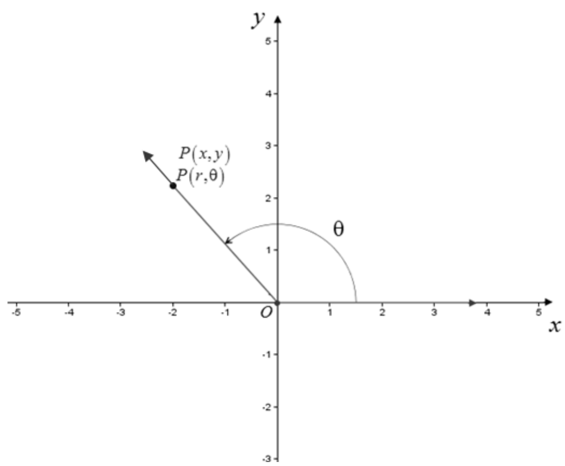
Note: A point located at the pole has coordinates $(0, \theta)$ where θ is **any** angle.

OBJECTIVE 3: Converting a Point from Polar Coordinates to Rectangular Coordinates

Relationships used when Converting a Point from Polar Coordinates to Rectangular Coordinates:

$$x = r\cos\theta$$

$$y = r\sin\theta$$

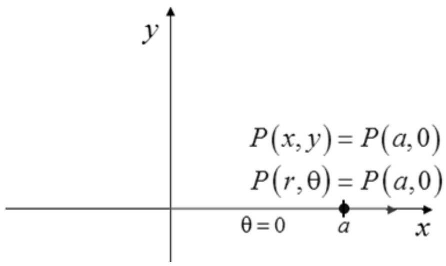


OBJECTIVE 4: Converting a Point from Rectangular Coordinates to Polar Coordinates

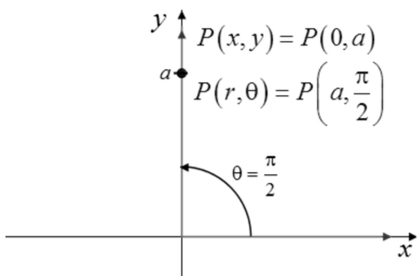
Converting Rectangular Coordinates to Polar Coordinates for Points Lying Along an Axis

In each case, assume that $a > 0$.

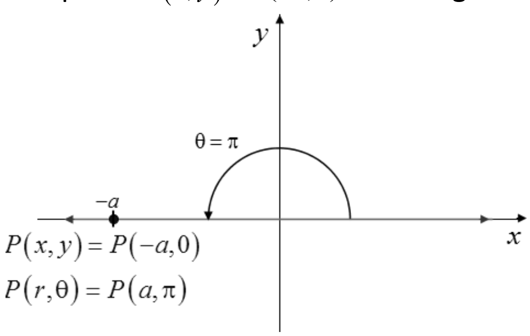
The point $P(x, y) = P(a, 0)$ lies along the positive *x*-axis and has polar coordinates of $P(r, \theta) = P(a, 0)$.



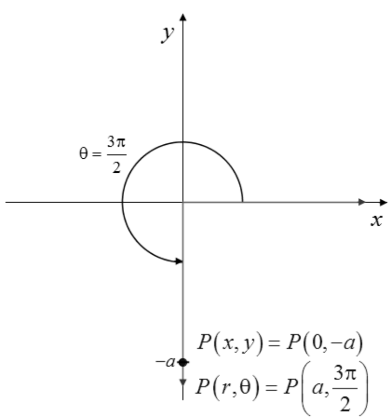
The point $P(x, y) = P(0, a)$ lies along the positive *y*-axis and has polar coordinates of $P(r, \theta) = P\left(a, \frac{\pi}{2}\right)$.



The point $P(x, y) = P(-a, 0)$ lies along the negative *x*-axis and has polar coordinates of $P(r, \theta) = P(a, \pi)$.

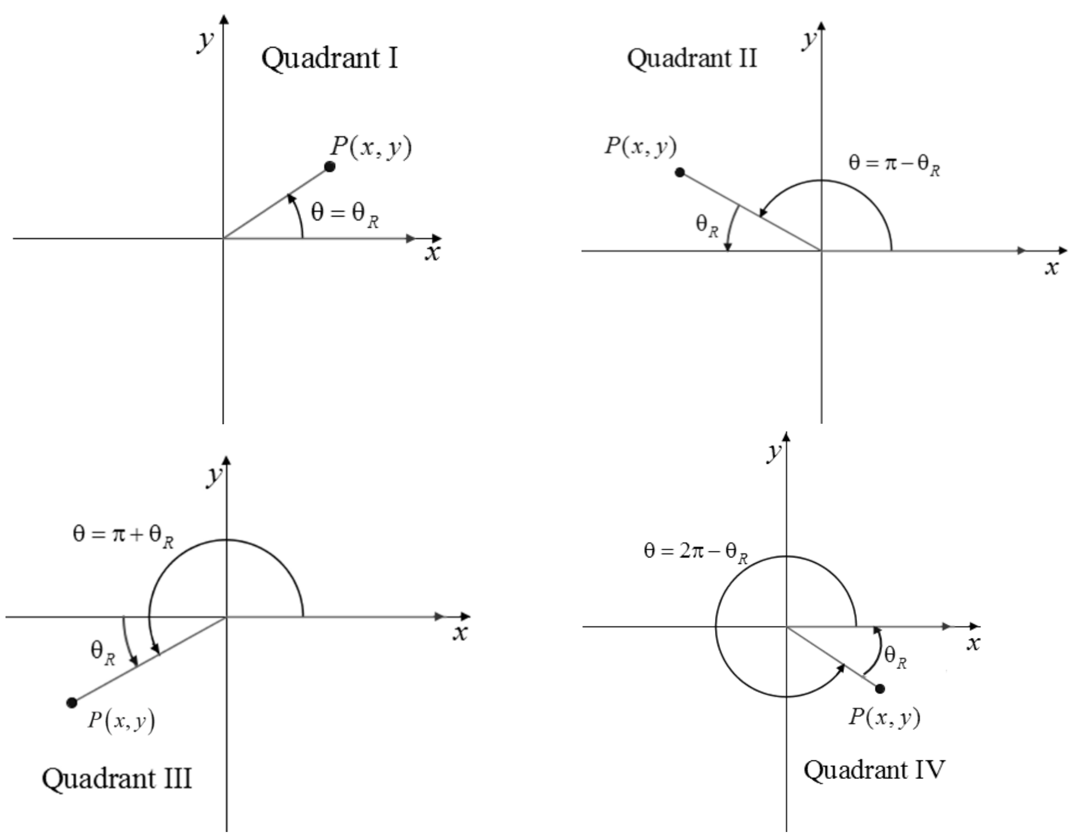


The point $P(x, y) = P(0, -a)$ lies along the negative *y*-axis and has polar coordinates of $P(r, \theta) = P\left(a, \frac{3\pi}{2}\right)$.



Converting Rectangular Coordinates to Polar Coordinates for Points Not Lying Along an Axis

1. Determine the value of *r* using the equation $r = \sqrt{x^2 + y^2}$.
2. Plot the point and determine the quadrant in which it lies.
3. Determine the value of the acute reference angle θ_R by solving the equation $\tan\theta_R = \left|\frac{y}{x}\right|$.
4. Determine the value of θ using θ_R and the quadrant in which the point lines. There are four cases:
    1) If $P(x, y)$ lies in Quadrant I, then $\theta = \theta_R$.
    2) If $P(x, y)$ lies in Quadrant II, then $\theta = \pi - \theta_R$.
    3) If $P(x, y)$ lies in Quadrant III, then $\theta = \theta_R + \pi$.
    4) If $P(x, y)$ lies in Quadrant IV, then $\theta = 2\pi - \theta_R$.



OBJECTIVE 5: Converting an Equation from Rectangular Form to Polar Form

A **polar equation** is an equation whose variables are *r* and θ. You will need to use the familiar relationships $x = r\cos\theta$, $y = r\sin\theta$, and $r^2 = x^2 + y^2$ to convert equations in *x* and *y* (rectangular form) to polar form.

OBJECTIVE 6: Converting an Equation from Polar Form to Rectangular Form

$x = r\cos\theta$ $\quad$ $y = r\sin\theta$ $\quad$ $r^2 = x^2 + y^2$ $\quad$ $\tan\theta = \dfrac{y}{x}$