Section 3.1 Relations and Functions

Objective 1: Understanding the Definitions of Relations and Functions

Definition: A **relation** is a correspondence between two sets *A* and *B* such that each element of set *A* corresponds to one or more elements of set *B.* Set *A* is called the **domain** of the relation and set *B* is called the **range** of the relation.

Definition: A **function** is a relation such that for each element in the domain, there corresponds *exactly one and only one* element in the range. In other words, a function is a well-defined relation.

The elements of the domain and range are typically listed in ascending order when using set notation.

Objective 2: Determine if Equations Represent Functions

To determine if an equation represents a function, we must show that for any value in the domain, there is exactly one corresponding value in the range.

Objective 3: Using Function Notation; Evaluating Functions

When an equation is explicitly solved for *y*, we say that "*y* is a function of *x*" or that the variable *y* depends on the variable *x*. Thus, *x* is the independent variable and *y* is the dependent variable.

CAUTION **The symbol $f(x)$ does not mean *f* times *x*. The notation $f(x)$ refers to the value of the function at *x*.**

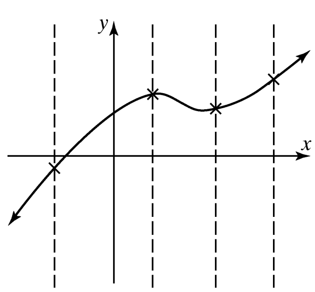
CAUTION **The expression $(-1)^2$ does not equal -1^2.**

The expression $\dfrac{f(x+h)-f(x)}{h}$ is called the **difference quotient** and is very important in calculus.

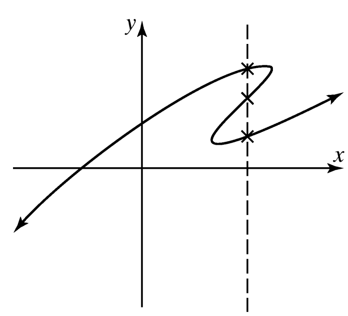
Objective 4: Using the Vertical Line Test

The Vertical Line Test

A graph in the Cartesian plane is the graph of a function if and only if no vertical line intersects the graph more than once.



This graph is a function.
(No vertical line intersects the graph more than once).



This graph is not a function.
(The graph does not pass the vertical line test).

Objective 5: Determining the Domain of a Function Given the Equation

The domain of a function $y = f(x)$ is the set of all values of *x* for which the function is defined.

It is very helpful to classify a function to determine its domain.

Definition: The function $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$ is a **polynomial function** of degree *n* where *n* is a nonnegative integer and $a_0, a_1, a_2, \ldots, a_n$ are real numbers. The domain of every polynomial function is $(-\infty, \infty)$.

Many functions can have restricted domains.

Definition: A **rational function** is a function of the form $f(x) = \dfrac{g(x)}{h(x)}$ where *g* and *h* are polynomial functions such that $h(x) \neq 0$. The domain of a rational function is the set of all real numbers such that $h(x) \neq 0$. If $h(x) = c$, where *c* is a real number, then we will consider the function $f(x) = \dfrac{g(x)}{h(x)} = \dfrac{g(x)}{c}$ to be a polynomial.

Definition: The function $f(x) = \sqrt[n]{g(x)}$ is a **root function** where *n* is a positive integer.

If *n* is *even*, the domain is the solution to the inequality $g(x) \geq 0$.

If *n* is *odd*, the domain is the set of all real numbers for which $g(x)$ is defined.