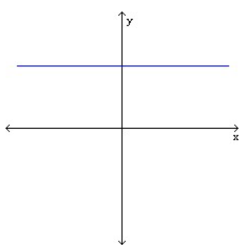
Section 3.3 Graphs of Basic Functions; Piecewise Functions

Objective 1: Sketching the Graphs of the Basic Functions

We begin by discussing the graphs of two specific linear functions. Recall that a linear function has the form $f(x) = mx + b$ where *m* is the slope of the line and *b* represents the *y*-coordinate of the *y*-intercept.

We start our discussion of the basic functions by looking at the **constant function**, that is, the linear function with $m = 0$, the graph of which is a horizontal line.

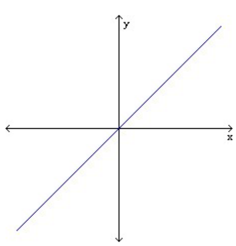
(1) The **Constant Function** $\boldsymbol{f(x) = b}$ has domain $(-\infty, \infty)$ and range $\{b\}$.



Notice that there are no arrows used at either end of the graph representing the constant function above. From this point forward in the text, unless the graph contains a definitive endpoint (shown by either an open dot or a closed dot) then it will be understood that the graph extends indefinitely in the same direction.

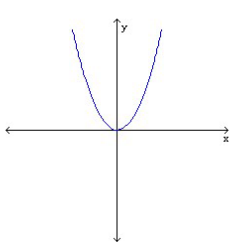
The **identity function** defined by $f(x) = x$ is another linear function with $m = 1$ and $b = 0$. It assigns to each number in the domain the exact same number in the range.

(2) The **Identity Function** $\boldsymbol{f(x) = x}$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$.



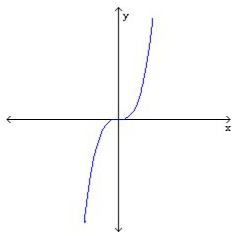
The **square function,** $f(x) = x^2$, assigns to each real number in the domain the square of that number in the range. The "u-shaped" graph of the square function is called a parabola.

(3) The **Square Function** $\boldsymbol{f(x) = x^2}$ has domain $(-\infty, \infty)$ and range $[0, \infty)$.



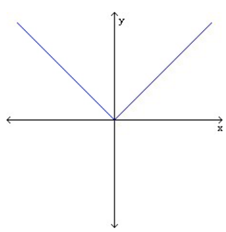
The **cube function**, $f(x) = x^3$, assigns to each real number in the domain the cube of that number in the range.

(4) The **Cube Function** $\boldsymbol{f(x) = x^3}$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$**.**



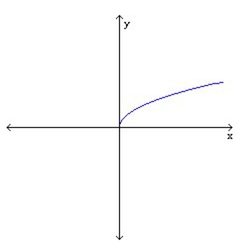
The **absolute value function**, $f(x) = |x|$, assigns to each real number in the domain the absolute value of that number in the range.

(5) The **Absolute Value Function** $\boldsymbol{f(x) = |x|}$ has domain $(-\infty, \infty)$ and range $[0, \infty)$.



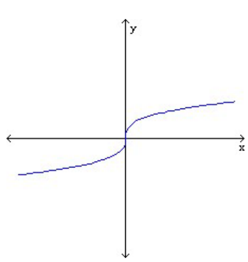
The **square root function**, $f(x) = \sqrt{x}$, is only defined for values of *x* that are greater than or equal to zero. It assigns to each real number in the domain the square root of that number in the range.

(6) The **Square Root Function** $\boldsymbol{f(x) = \sqrt{x}}$ has domain $[0, \infty)$ and range $[0, \infty)$.



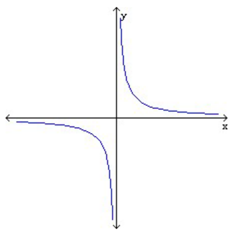
Unlike the square root function which is only defined for values of *x* greater than or equal to zero, the **cube root function,** $f(x) = \sqrt[3]{x}$ **,** is defined for all real numbers and assigns to each number in the domain the cube root of that number in the range.

(7) The **Cube Root Function** $\boldsymbol{f(x) = \sqrt[3]{x}}$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$**.**



The **reciprocal function**, $f(x)=\frac{1}{x}$, is a rational function whose domain is $\{x\,|\,x\neq 0\}$. It assigns to each number *a* in the domain its reciprocal, $\frac{1}{a}$, in the range. The reciprocal function has two asymptotes. The *y*-axis (the line $x=0$) is a vertical asymptote and the *x*-axis (the line $y=0$) is a horizontal asymptote**.**

(8) The **Reciprocal Function** $\boldsymbol{f(x)=\frac{1}{x}}$ has domain $(-\infty,0)\cup(0,\infty)$ and range $(-\infty,0)\cup(0,\infty)$.

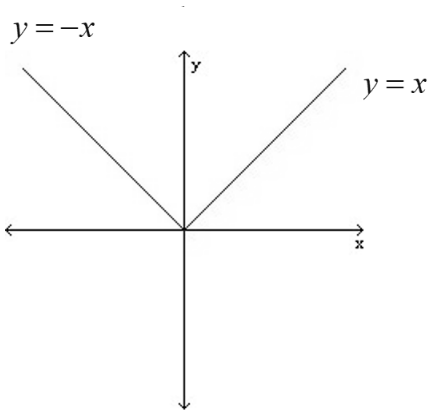


Objective 2: Sketching the Graphs of Basic Functions with Restricted Domains

Objective 3: Analyzing Piecewise Defined Functions

The absolute value function, $f(x)=|x|$, can also be defined by a rule that has two different "pieces."

$$f(x)=|x|=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$$



You can see by the graph above that the "left-hand piece" is actually a part of the line $y=-x$ while the "right-hand piece" is a part of the line $y=x$.

Functions defined by a rule that has more than one "piece" are called piecewise-defined functions.