Section 3.4  Transformations of Functions

Objective 1: Using Vertical Shifts to Graph Functions

Let $c$ be a positive real number.

1. The graph of $y = f(x) + c$ is obtained by shifting the graph of $y = f(x)$ vertically upward $c$ units.
2. The graph of $y = f(x) - c$ is obtained by shifting the graph of $y = f(x)$ vertically downward $c$ units.

Objective 2: Using Horizontal Shifts to Graph Functions

Let $c$ be a positive real number.

1. The graph of $y = f(x + c)$ is obtained by shifting the graph of $y = f(x)$ horizontally to the left $c$ units.
2. The graph of $y = f(x - c)$ is obtained by shifting the graph of $y = f(x)$ horizontally to the right $c$ units.

For $c > 0$, the graph of $y = f(x - c)$ is the graph of $f$ shifted to the right $c$ units. At first glance, it appears that the rule for horizontal shifts is the opposite of what seems natural. Substituting $x + c$ for $x$ causes the graph of $y = f(x)$ to be shifted to the left while substituting $x - c$ for $x$ causes the graph to shift to the right $c$ units.
Objective 3: Using Reflections to Graph Functions

The graph of \( y = -f(x) \) is a reflection of the graph of \( y = f(x) \) about the x-axis.

The graph of \( y = f(-x) \) is a reflection of the graph of \( y = f(x) \) about the y-axis.

Objective 4: Using Vertical Stretches and Compressions to Graph Functions

Suppose \( a \) is a positive real number. The graph of \( y = af(x) \) is obtained by multiplying each y-coordinate of \( y = f(x) \) by \( a \).

1. If \( a > 1 \), the graph of \( y = af(x) \) is a vertical stretch of the graph of \( y = f(x) \).
2. If \( 0 < a < 1 \), the graph of \( y = af(x) \) is a vertical compression of the graph of \( y = f(x) \).
Objective 6: Using Combinations of Transformations to Graph Functions

When graphing a function that involves multiple transformations, it is important to follow a certain “order of operations.” In our text, transformations are performed in the following order:

1. Horizontal shifts
2. Reflection about y-axis
3. Vertical stretches/compressions
4. Reflection about x-axis
5. Vertical shifts

Although different ordering is possible, the order above will always work.

Objective 7: Using Transformations to Sketch the Graphs of Piecewise-Defined Functions