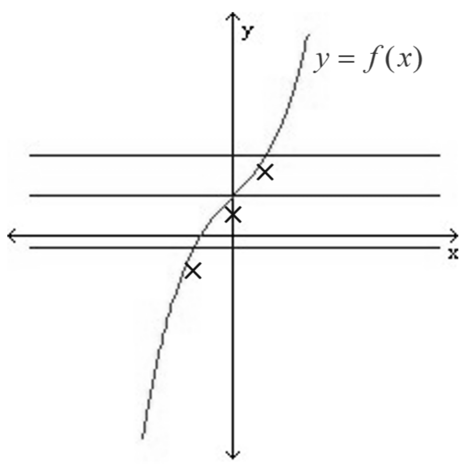
Section 3.6 One-to-one Functions; Inverse Functions

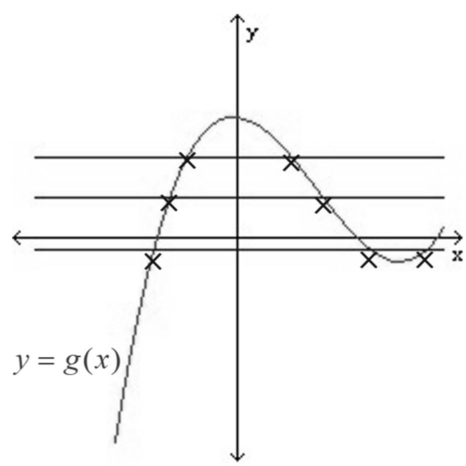
Objective 1: Understanding the Definition of a One-to-one Function

Definition: A function *f* is **one-to-one** if for any values $a \neq b$ in the domain of *f*, $f(a) \neq f(b)$.

Interpretation: For $f(x) = y$ to be a function, we know that for each *x* in the domain there exists one and only one *y* in the range. For $f(x) = y$ to be a one-to-one function, both of the following must be true: for each *x* in the domain there exists one and only one *y* in the range, AND for each *y* in the range there exists one and only one *x* in the domain.

Objective 2: Determining if a Function is One-to-one Using the Horizontal Line Test



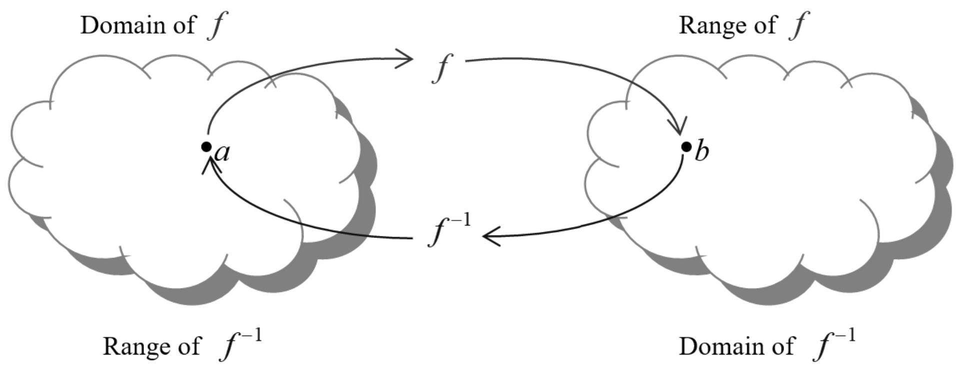


The Horizontal Line Test
If every horizontal line intersects the graph of a function *f* at most once, then *f* is one-to-one.

Objective 3: Understanding and Verifying Inverse Functions

Every one-to-one function has an inverse function.

Definition: Let *f* be a one-to-one function with domain *A* and range *B*. Then f^{-1} is **the inverse function of *f*** with domain *B* and range *A*. Furthermore, if $f(a)=b$ then $f^{-1}(b)=a$.



Do not confuse f^{-1} with $\frac{1}{f(x)}$. The negative 1 in f^{-1} is NOT an exponent!

Inverse functions "undo" each other.

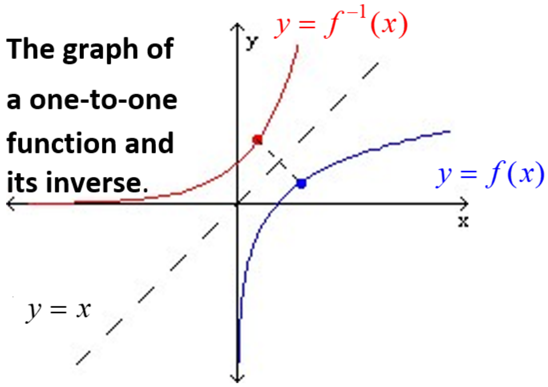
Composition Cancellation Equations:

$f\left(f^{-1}(x)\right)=x$ for all *x* in the domain of f^{-1}

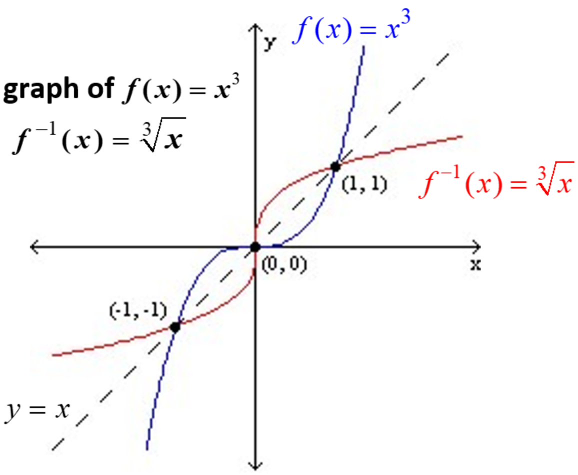
$f^{-1}\left(f(x)\right)=x$ for all *x* in the domain of *f*

Objective 4: Sketching the Graphs of Inverse Functions

The graph of f^{-1} is a reflection of the graph of *f* about the line $y = x$.
If the functions have any points in common, they must lie along the line $y = x$.



The graph of a one-to-one function and its inverse.



The graph of $f(x) = x^3$ and $f^{-1}(x) = \sqrt[3]{x}$

Objective 5: Finding the Inverse of a One-to-one Function

We know that if a point (x, y) is on the graph of a one-to-one function, then the point (y, x) is on the graph of its inverse function.

To find the inverse of a one-to-one function, replace $f(x)$ with *y,* interchange the variables *x* and *y*, and then solve for *y.* This is the function $f^{-1}(x)$.

Inverse Function Summary

1. The inverse function f^{-1} exists if and only if the function *f* is one-to-one.
2. The domain of *f* is the same as the range of f^{-1} and the range of *f* is the same as the domain of f^{-1}.
3. To verify that two one-to-one functions *f* and *g* are inverses of each other, use the composition cancellation equations to show that $f\left(g(x)\right) = g\left(f(x)\right) = x$.
4. The graph of f^{-1} is a reflection of the graph of *f* about the line $y = x$. That is, for any point $\left(a, b\right)$ that lies on the graph of *f,* the point $\left(b, a\right)$ must lie on the graph of f^{-1}.
5. To find the inverse of a one-to-one function, replace $f(x)$ with *y,* interchange the variables *x* and *y*, and then solve for *y.* This is the function $f^{-1}(x)$.