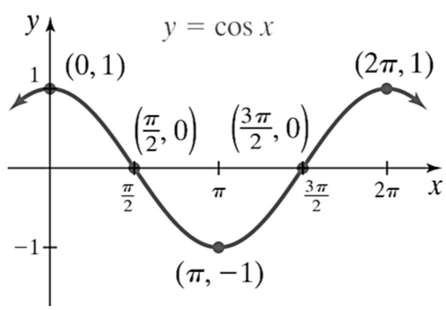
7.2a More on the Graphs of Sine and Cosine: Phase Shift

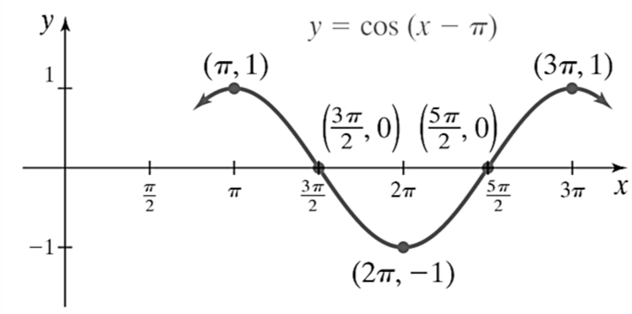
OBJECTIVE 1: Sketching Functions of the Form $y = \sin(x - C)$ and $y = \cos(x - C)$

The graph of $y = \sin(x - C)$ is **a horizontal shift** of the graph of $y = \sin x$, and the graph of $y = \cos(x - C)$ is a horizontal shift of the graph of $y = \cos x$.

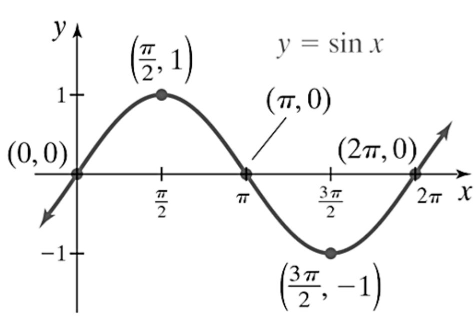
If $C > 0$, the shift is *C* units to the right, but if $C < 0$, the shift is *C* units to the left.

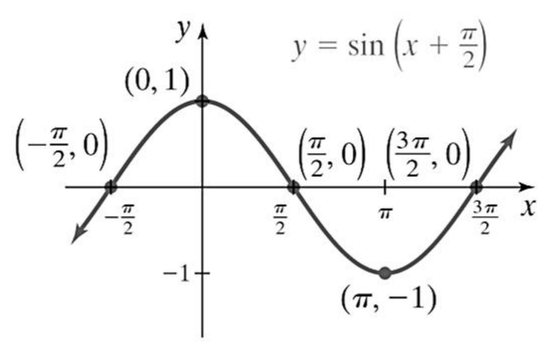
Consider the function $y = \cos(x - \pi)$. Since $C = \pi$ and $\pi > 0$, the graph is shifted to the right π units. The quarter points for $y = \cos(x - \pi)$ are obtained by adding π to the *x*-coordinate of each quarter point of $y = \cos x$. The graph of $y = \cos x$ is shown below on the left, and the graph of $y = \cos(x - \pi)$ is shown below on the right.





Now, consider the function $y = \sin\left(x + \frac{\pi}{2}\right)$. Since $C = -\frac{\pi}{2}$ and $-\frac{\pi}{2} < 0$, the graph is shifted to the left $\frac{\pi}{2}$ units. The quarter points for $y = \sin\left(x + \frac{\pi}{2}\right)$ are obtained by subtracting $\frac{\pi}{2}$ from the *x*-coordinate of each quarter point of $y = \sin x$. The graph of $y = \sin x$ is shown below on the left, and the graph of $y = \sin\left(x + \frac{\pi}{2}\right)$ is shown below on the right.





OBJECTIVE 2: Sketching Functions of the Form $y = A\sin(Bx - C)$ and $y = A\cos(Bx - C)$

Horizontal stretching and horizontal shifting of sine or cosine functions together determine the **phase shift** of the function. The formula for phase shift is $\frac{C}{B}$, $B > 0$, and this value will be the *x*-coordinate of the first quarter point of the graph of the function $y = A\sin(Bx - C)$, $B > 0$ or $y = A\cos(Bx - C)$, $B > 0$.

Steps for Sketching Functions of the Form $y = A\sin(Bx - C)$ and $y = A\cos(Bx - C)$

1. Rewrite the function as $y = A\sin\left(B\left(x - \frac{C}{B}\right)\right)$ or $y = A\cos\left(B\left(x - \frac{C}{B}\right)\right)$. If $B < 0$, then use the even and odd properties of the sine and cosine function to write the function in an equivalent form such that $B > 0$.

   We now use this new form to determine the amplitude, period, and phase shift.

2. The amplitude is $|A|$. The range is $\left[-|A|, |A|\right]$.

3. The period is $P = \frac{2\pi}{B}$.

4. The phase shift is $\frac{C}{B}$.

5. The *x*-coordinate of the first quarter point is $\frac{C}{B}$. The *x*-coordinate of the last quarter point is $\frac{C}{B} + P$. An interval for one complete cycle is $\left[\frac{C}{B}, \frac{C}{B} + P\right]$. Subdivide this interval into 4 equal subintervals of length $P \div 4$ by starting with $\frac{C}{B}$ and adding $(P \div 4)$ to the *x*-coordinate of each successive quarter point.

6. Multiply the *y*-coordinates of the quarter points of $y = \sin x$ or $y = \cos x$ by *A* to determine the *y*-coordinates of the corresponding quarter points for $y = A\sin(Bx - C) = A\sin\left(B(x - \frac{C}{B})\right)$ and $y = A\cos(Bx - C) = A\cos\left(B(x - \frac{C}{B})\right)$.

7. Connect the quarter points to obtain one complete cycle.