# 7.2b More on the Graphs of Sine and Cosine: Vertical Shift

**OBJECTIVE 3: Sketching Functions of the Form** $y = A\sin(Bx - C) + D$ **and** $y = A\cos(Bx - C) + D$

The "+ *D*" added to the functions we have been graphing causes a **vertical shift** of the graph.

**If** $D > 0$ **, the shift is *D* units up, but if** $D < 0$ **, the shift is *D* units down.**

**Steps for Sketching Functions of the Form** $y = A\sin(Bx - C) + D$ **and** $y = A\cos(Bx - C) + D$

1.  Rewrite the function as $y = A\sin\left(B\left(x - \dfrac{C}{B}\right)\right) + D$ or $y = A\cos\left(B\left(x - \dfrac{C}{B}\right)\right) + D$. If $B < 0$,

    then use the even and odd properties of the sine and cosine function to write the function in an equivalent form such that $B > 0$.

    **We now use this new form to determine the amplitude, period, and phase shift.**

2.  The amplitude is $|A|$. The range is $\left[-|A| + D, \, |A| + D\right]$.

3.  The period is $P = \dfrac{2\pi}{B}$.

4.  The phase shift is $\dfrac{C}{B}$.

5.  The *x*-coordinate of the first quarter point is $\dfrac{C}{B}$. The *x*-coordinate of the last quarter point is

    $\dfrac{C}{B} + P$. An interval for one complete cycle is $\left[\dfrac{C}{B}, \dfrac{C}{B} + P\right]$. Subdivide this interval into 4

    equal subintervals of length $P \div 4$ by starting with $\dfrac{C}{B}$ and adding $(P \div 4)$ to the *x*-

    coordinate of each successive quarter point.

6.  Multiply the *y*-coordinates of the quarter points of $y = \sin x$ or $y = \cos x$ by *A* and then add
    *D* to determine the *y*-coordinates of the corresponding quarter points for
    $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$.

7.  Connect the quarter points to obtain one complete cycle.