7.2b More on Graphs of Sine and Cosine: Vertical Shift

OBJECTIVE 3: Sketching Graphs of the Form $\boldsymbol{y = A\sin\left(Bx - C\right) + D}$ and $\boldsymbol{y = A\cos\left(Bx - C\right) + D}$

The "+ *D*" added to the functions we have been graphing causes a **vertical shift** of the graph.
If $D > 0$, the shift is *D* units up, but if $D < 0$, the shift is *D* units down.

Steps for Sketching Functions of the Form $\boldsymbol{y = A\sin\left(Bx - C\right) + D}$ and $\boldsymbol{y = A\cos\left(Bx - C\right) + D}$

1. Rewrite the function as $y = A\sin\left(B\left(x - \frac{C}{B}\right)\right) + D$ or $y = A\cos\left(B\left(x - \frac{C}{B}\right)\right) + D$. If $B < 0$, then use the even and odd properties of the sine and cosine function to write the function in an equivalent form such that $B > 0$.

   We now use this new form to determine the amplitude, period, and phase shift.

2. The amplitude is $|A|$. The range is $\left[-|A| + D, |A| + D\right]$.
3. The period is $P = \frac{2\pi}{B}$.
4. The phase shift is $\frac{C}{B}$.
5. The *x*-coordinate of the first quarter point is $\frac{C}{B}$. The *x*-coordinate of the last quarter point is $\frac{C}{B} + P$. An interval for one complete cycle is $\left[\frac{C}{B}, \frac{C}{B} + P\right]$. Subdivide this interval into 4 equal subintervals of length $P \div 4$ by starting with $\frac{C}{B}$ and adding $\left(P \div 4\right)$ to the *x*-coordinate of each successive quarter point.
6. Multiply the *y*-coordinates of the quarter points of $y = \sin x$ or $y = \cos x$ by *A* and then add *D* to determine the *y*-coordinates of the corresponding quarter points for $y = A\sin\left(Bx - C\right) + D$ and $y = A\cos\left(Bx - C\right) + D$.
7. Connect the quarter points to obtain one complete cycle.

OBJECTIVE 4: Determine the Equation of a Function of the Form $y = A\sin\left(Bx - C\right) + D$ or $y = A\cos\left(Bx - C\right) + D$ Given Its Graph