8.1 Trigonometric Identities

OBJECTIVE 1: Reviewing the Fundamental Identities

The Quotient Identities: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

The Reciprocal Identities: $\sin\theta = \dfrac{1}{\csc\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

The Pythagorean Identities: $\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

The Odd Properties: $\sin(-\theta) = -\sin\theta$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\cot(-\theta) = -\cot\theta$

The Even Properties: $\cos(-\theta) = \cos\theta$

$\sec(-\theta) = \sec\theta$

OBJECTIVE 2: Substituting Known Identities to Verify an Identity

OBJECTIVE 3: Changing to Sines and Cosines to Verify an Identity

OBJECTIVE 4: Factoring to Verify an Identity

Difference of Two Squares $a^2 - b^2 = (a+b)(a-b)$

Perfect Square Formulas $a^2 + 2ab + b^2 = (a+b)^2$ and $a^2 - 2ab + b^2 = (a-b)^2$

OBJECTIVE 5: Separating a Single Quotient into Multiple Quotients to Verify an Identity

OBJECTIVE 6: Combining Fractional Expressions to Verify an Identity

OBJECTIVE 7: Multiplying by Conjugates to Verify Identities

OBJECTIVE 8: Summarizing the Techniques for Verifying Identities

First, start with what appears to be the more difficult side of the given identity. Try to transform this side so that it eventually matches identically into the other side. Do not hesitate to start over and work with the other side of the identity if you are having trouble. Try using one of the following techniques to begin, and then use others, if necessary, to continue the verification process.

1. Look for ways to use a known identity such as the reciprocal identities, quotient identities, and even/odd properties. If the identity includes a squared trigonometric expression, try using a variation of a Pythagorean identity.
2. Try rewriting each trigonometric expression in terms of sines and cosines.
3. Factor out a greatest common factor and use algebraic factoring techniques such as factoring the difference of two squares or factoring perfect squares.
4. If a single term appears in a quotient, try separating the quotient into two or more terms: $\frac{A+B}{C}=\frac{A}{C}+\frac{B}{C}$.
5. If there are two or more fractional expressions, try combining the expressions using a common denominator: $\frac{A}{B}+\frac{C}{D}=\frac{AD+BC}{BD}$.
6. If the numerator or denominator of one or more quotients contains an expression of the form $A+B$, try multiplying the numerator and denominator by its conjugate $A-B$: $\frac{C}{A+B}=\frac{C}{A+B}\cdot\frac{A-B}{A-B}$ or $\frac{A+B}{C}=\frac{A+B}{C}\cdot\frac{A-B}{A-B}$.